Integrated Multiuser Reception
& Vector Channel Estimation

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First Interim Progress Report

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Abstract

In this report the use of a crossed-dipole array is proposed in joint space-time channel estimation for asynchronous multipath DS-CDMA systems. The polarisation diversity offered by such an array, unlike linearly polarised arrays, is able to detect and estimate any arbitrary completely polarised signal path. By utilising the polarisation information inherent in the received signal to construct the polar-spatio-temporal array (polar-STAR) manifold vector, the accuracy and resolution of the polar-STAR parameters' estimation are significantly improved, and its signal detection capability is enhanced. To alleviate the need for a multidimensional search in the polarisation space, a computationally efficient joint polarisation-angle-delay channel parameter estimation algorithm is proposed for a 'desired user' that operates in an asynchronous multi-user and multipath environment. The proposed algorithm, which can be seen as an application of MUSIC-type techniques, is based on combining a two-dimensional STAR-Subspace type technique with a set of analytical equations and is supported by representative examples and computer simulation studies.
Acknowledgements

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Mathematical Notation

$A$ scalar

$\mathbf{A}$ vector

$A$ matrix

$\mathbb{I}_N$ $N \times N$ identity matrix

$0_N$ zero vector of $N$ elements

$A^b$ element by element power

$\text{arg}(A)$ argument of $A$

$\text{exp}(A)$ element by element exponential of vector $A$

$\text{diag}(A)$ diagonalisation of vector $A$

$\text{vec}(A)$ rowwise vectorisation of matrix $A$ into a single long column vector

$\text{det}(A)$ determinant of matrix $A$

$\text{trace}(A)$ trace of matrix $A$

$\text{exp}(\bullet)$ elementwise exponential

$(\bullet)^*$ complex conjugate

$(\bullet)^T$ transpose

$(\bullet)^H$ Hermitian transpose

$\lfloor \bullet \rfloor$ round up to integer

$\odot$ Hadamard (Schur) product

$\otimes$ Kronecker product

$\mathbb{N}$ the set of natural numbers

$\mathbb{R}$ the field of real numbers

$\mathbb{C}$ the field of complex numbers
# List of Acronyms

<table>
<thead>
<tr>
<th>Acronym</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>AWGN</td>
<td>additive white Gaussian noise</td>
</tr>
<tr>
<td>CDMA</td>
<td>code division multiple access</td>
</tr>
<tr>
<td>COLD</td>
<td>co-centered orthogonal loop and dipole</td>
</tr>
<tr>
<td>DS-CDMA</td>
<td>direct sequence code division multiple access</td>
</tr>
<tr>
<td>DS-SS</td>
<td>direct sequence spread spectrum</td>
</tr>
<tr>
<td>DOA</td>
<td>direction of arrival</td>
</tr>
<tr>
<td>IS-95</td>
<td>Interim Standard 95</td>
</tr>
<tr>
<td>ISI</td>
<td>intersymbol interference</td>
</tr>
<tr>
<td>ITU</td>
<td>International Telecommunication Union</td>
</tr>
<tr>
<td>MAI</td>
<td>multiple-access interference</td>
</tr>
<tr>
<td>ML</td>
<td>maximum likelihood</td>
</tr>
<tr>
<td>MLSE</td>
<td>maximum likelihood sequence estimation</td>
</tr>
<tr>
<td>MMSE</td>
<td>minimum mean square error</td>
</tr>
<tr>
<td>MUSIC</td>
<td>multiple signal classification</td>
</tr>
<tr>
<td>PN</td>
<td>pseudo-noise</td>
</tr>
<tr>
<td>PADE</td>
<td>polarisation-angle-delay estimation</td>
</tr>
<tr>
<td>POLAR</td>
<td>polarisation</td>
</tr>
<tr>
<td>POLAR-STAR</td>
<td>polar-spatio-temporal array</td>
</tr>
<tr>
<td>SIR</td>
<td>signal to interference ratio</td>
</tr>
<tr>
<td>SNR</td>
<td>signal to noise ratio</td>
</tr>
<tr>
<td>SS</td>
<td>spread spectrum</td>
</tr>
<tr>
<td>ST</td>
<td>spatial-temporal</td>
</tr>
<tr>
<td>STAR</td>
<td>space-time array</td>
</tr>
<tr>
<td>TEM</td>
<td>transverse electromagnetic</td>
</tr>
<tr>
<td>TDL</td>
<td>tapped-delay line</td>
</tr>
<tr>
<td>TOA</td>
<td>time of arrival</td>
</tr>
<tr>
<td>W-CDMA</td>
<td>wideband code division multiple access</td>
</tr>
<tr>
<td>2G / 3G</td>
<td>second generation / third generation</td>
</tr>
</tbody>
</table>
# List of Symbols

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Definition</th>
</tr>
</thead>
<tbody>
<tr>
<td>$n$</td>
<td>$n^{th}$ bit or symbol</td>
</tr>
<tr>
<td>$i$</td>
<td>$i^{th}$ user</td>
</tr>
<tr>
<td>$j$</td>
<td>$j^{th}$ path</td>
</tr>
<tr>
<td>$m$</td>
<td>$m^{th}$ chip</td>
</tr>
<tr>
<td>$a_i[n]$</td>
<td>bit</td>
</tr>
<tr>
<td>$\alpha_i[m]$</td>
<td>PN-code chip</td>
</tr>
<tr>
<td>$\tilde{x}[n]$</td>
<td>long stacked received data vector</td>
</tr>
<tr>
<td>$\tilde{n}[n]$</td>
<td>discrete AWGN</td>
</tr>
<tr>
<td>$\tilde{x}(t)$</td>
<td>received complex baseband signal</td>
</tr>
<tr>
<td>$m_c(t)$</td>
<td>complex baseband DS-CDMA signal</td>
</tr>
<tr>
<td>$n(t)$</td>
<td>AWGN</td>
</tr>
<tr>
<td>$c_{PN,i}(t)$</td>
<td>PN spreading waveform</td>
</tr>
<tr>
<td>$c(t)$</td>
<td>chip pulse shaping waveform</td>
</tr>
<tr>
<td>$p(\gamma, \eta)$</td>
<td>signal's state of polarisation</td>
</tr>
<tr>
<td>$u(\theta, \phi)$</td>
<td>unit vector</td>
</tr>
<tr>
<td>$k(\theta, \phi)$</td>
<td>wavenumber vector</td>
</tr>
<tr>
<td>$\Omega(\theta, \phi)$</td>
<td>spherical-to-Cartesian transformation matrix</td>
</tr>
<tr>
<td>$M(\theta)$</td>
<td>horizontal and vertical spatial array manifold vector</td>
</tr>
<tr>
<td>$q(\Theta)$</td>
<td>electric field components vector</td>
</tr>
<tr>
<td>$\theta, \phi$</td>
<td>azimuth and elevation directions</td>
</tr>
<tr>
<td>$\gamma, \eta$</td>
<td>polarisation parameters</td>
</tr>
<tr>
<td>$T_{cs}$</td>
<td>symbol period</td>
</tr>
<tr>
<td>$T_c$</td>
<td>chip period</td>
</tr>
<tr>
<td>$\tau_{ij}$</td>
<td>path delay</td>
</tr>
<tr>
<td>$l_{ij}$</td>
<td>discrete path delay in multiples of $T_c$</td>
</tr>
<tr>
<td>$C$</td>
<td>mutual coupling matrix</td>
</tr>
<tr>
<td>$F$</td>
<td>Fourier transformation matrix</td>
</tr>
<tr>
<td>$E_n$</td>
<td>noise eigenvectors matrix</td>
</tr>
<tr>
<td>$A$</td>
<td>polar-spatial array manifold vector</td>
</tr>
<tr>
<td>$S$</td>
<td>spatial array manifold vector</td>
</tr>
<tr>
<td>$\mathcal{V}$</td>
<td>sensor's sensitivities</td>
</tr>
<tr>
<td>$\mathcal{Z}_{DP}$</td>
<td>$D^{th}$ user's preprocessor</td>
</tr>
<tr>
<td>$N$</td>
<td>number of antennas in the base station array</td>
</tr>
<tr>
<td>$N_c$</td>
<td>number of chips per symbol</td>
</tr>
<tr>
<td>$M$</td>
<td>number of users</td>
</tr>
<tr>
<td>$K_i$</td>
<td>number of multipath rays</td>
</tr>
<tr>
<td>$\xi_i$</td>
<td>$i^{th}$ user's PN-code sequence</td>
</tr>
<tr>
<td>$\Theta_{ij}$</td>
<td>path's parameter vector</td>
</tr>
<tr>
<td>$h_{ij}$</td>
<td>polar-space-time manifold vector</td>
</tr>
<tr>
<td>$\beta_{ij}$</td>
<td>complex path coefficient</td>
</tr>
</tbody>
</table>
1 Introduction

This section outlines the integration of the two technologies: DS-CDMA and space-time array processing in the realisation of the space-time array communication systems, which is subsequently extended to polarisation-space-time communication system which is the main focus of this report. The section is finally concluded with the organisation of the report.

1.1 Polarisation-Space-Time Architecture

In wireless communications, the signal emitted by a mobile terminal normally suffers multiple reflections and scattering along the transmission path, hence creating several replicas with different arriving angle, path delay, polarisation and fading. However, among these four channel parameters, the polarisation factor of the signal, which describe the orientation of its electric field, often receives disproportionately little attention in traditional model of array processing. Most array processing techniques assume the employment of polarisation-insensitive sensors, which therefore presume that the polarisation of the received signal is perfectly aligned with respect to the orientation of the sensors, thus obviating any polarisation mismatches. But in typical mobile environments, the received signal rarely takes on its transmitted polarisation due to the depolarisation mechanism [2] intrinsical in the propagation channel (especially in an urban environment). This is further aggravated by the frequent random angular orientation of most portable handheld devices. Such diversity in signal's polarisation, which is normally treated as part of signal fading (i.e. polarisation fading), can be exploited to provide an extra degree of signal discrimination, and as such improve the receiver's detection/estimation capabilities. Note that the polarisation state of the signal can be either completely or partially polarised, but as with many studies, the main focus of this work will assume complete polarisation. Investigation on partially polarised scenario can be found in [3, 4].
Polarisation diversity has been studied in a number of direction finding algorithms to ameliorate its angle-of-arrival estimation [5-11]. This is achieved by means of diversely polarised arrays which are sensitive to the polarisation of the received signal. Ferrara and Parks [5] have shown that by employing an array of diversely polarised sensors, the angle-of-arrival estimation is significantly improved as multiple signals can now be resolved on the basis of polarisations in addition to their arriving angles. Schmidt [6], on the other hand, demonstrated the ability of distinguishing two highly correlated signals by incorporating their signal polarisations. Since then, various diversely polarised arrays have been proposed in a correlated signal environment. For instance, in [7] a diversely polarised array consisting of circularly-polarised sensors is used with the Cramer-Rao bound to evaluate the angle estimation accuracy of correlated signals. In [8], a linear array of crossed dipoles, which measures its horizontal and vertical responses separately, is used with the ESPRIT algorithm to estimate the angle and polarisation of coherent signals. The performance of the angle and polarisation estimation is then further improved in [9] by using a co-centered orthogonal loop and dipole (COLD) array. Other diversely polarised arrays include, for instance, the use of dipole triad and/or loop triad(s) for multisource angle and polarisation estimation [12]. Note that a dipole, or loop, triad consists of three identical and orthogonally colocated electrically short dipoles, or magnetically small loops, respectively. In [13], the concept of 'electromagnetic vector sensor' is employed for self-initiating MUSIC-based direction finding and polarisation estimation algorithm in the spatio-polarisational beamspace. An electromagnetic vector sensor is comprised of six spatially colocated nonidentical nonisotropic antennas where the signal's three electric-field components and three magnetic-field components are each measured separately. These electromagnetic vector sensors were also employed in [14] for ESPIRT-based blind beamforming or geolocation of wideband fast frequency-hopping signals. However, little work [15, 25, 17] has been done to include polarisation in the area of joint space-time channel estimation [18],
especially in a correlated signal environment. Hence in this study, a novel joint direction of arrival and time delay estimation approach is proposed for asynchronous DS-CDMA systems, in conjunction with analytical expressions providing the estimate of the polarisation parameters. The proposed approach is a subspace-type method having superresolution capabilities. However, it requires knowledge of the Array Manifold, which implies that the array should be properly calibrated.

1.2 Organisation of Report

To realistically model the multipath channel using polarisation-sensitive sensors, instead of using the traditional isotropic sensors, a generalised signal model is formulated as detailed in Section 2. The well-known spatial array manifold vector commonly-used for polarisation-insensitive array is first extended to include the polarisation element to form its corresponding manifold vector. The temporal dimension is then incorporated to construct the polar-spatio-temporal array (polar-STAR) manifold vector for the asynchronous DS-CDMA systems. Based on the formulation, a subspace-based polarisation-angle-delay estimation (PADE) algorithm is then proposed in Section 3, together with a novel temporal smoothing technique which restores the desired subspace dimensionality. Following that, Section 4 provides several simulation studies which depict the performance of the proposed algorithm. The report is finally concluded in Section 5, together with the future directions of the research.
2 Signal Model

In this section an asynchronous space-time polarisation-sensitive array communication system is described. The signal received at the base station array is modelled from the well-known isotropic sensor array down to the polarisation-sensitive sensor array perspective. The model is simplified to a simple and concise expression which is later employed in the remaining sections of the report.

2.1 Sensor Array Manifold Vector

Consider an antenna array of $N$ polarisation-insensitive sensors, its corresponding spatial array manifold vector due to a signal path arriving from the direction $(\theta, \phi)$, with $\theta$ and $\phi$ representing the azimuth and elevation directions respectively, can be expressed as

$$S(\theta, \phi) = \exp\left(-j{r_x, r_y, r_z}k(\theta, \phi)\right)$$

(1)

where $[r_x, r_y, r_z]^T$ is a $3 \times N$ matrix denoting the Cartesian coordinates of the sensors, and $k(\theta, \phi) = (2\pi/\lambda).u(\theta, \phi)$ is the wavenumber vector with the unit vector $u(\theta, \phi) = [\cos(\theta)\cos(\phi), \sin(\theta)\cos(\phi), \sin(\phi)]^T$.

This can be easily extended to the polarisation-sensitive array manifold vector [7] by specifying the polarisation of the signal path. Thus consider the case of an array of $N$ tripoles [19] sensors (i.e. sets of three co-centered orthogonal dipoles with the signal from each dipole being processed separately in the array). Given a completely polarised transverse electromagnetic (TEM) signal impinging onto the array, its polarisation ellipse [20] generated by the electric field can be uniquely described by the parameters $\gamma$ and $\eta$, where $0 \leq \gamma \leq \pi/2$ and $-\pi \leq \eta < \pi$. Hence for a given path of arbitrary elliptical polarisation propagating into the array, its associated manifold vector can be derived as

$$A(\Theta) = S(\theta, \phi) \otimes q(\Theta)$$

(2)

where $\Theta \triangleq [\theta, \phi, \gamma, \eta]^T$ is the path’s parameter vector, $\otimes$ is the Kronecker product,
and \( \mathbf{q}(\Theta) = [E_x(\Theta), E_y(\Theta), E_z(\Theta)]^T \) is a 3-dimensional vector containing the electric field components induced on each dipole, that is given by

\[
\mathbf{q}(\Theta) = \text{diag}(\mathcal{V}) \cdot \mathcal{T}(\theta, \phi) \cdot \mathcal{P}(\gamma, \eta)
\]

(3)

where \( \mathcal{V} = [V_x, V_y, V_z]^T \) is the sensor's sensitivities to unit electric fields solely polarised in the \( x, y, \) and \( z \) directions respectively, \( \mathcal{T}(\theta, \phi) = \{1/\cos(\phi)\}, \partial \mathbf{u} / \partial \theta, \partial \mathbf{u} / \partial \phi \} \) is the Spherical-to-Cartesian transformation matrix, and \( \mathcal{P}(\gamma, \eta) = [\cos(\gamma), \sin(\gamma) e^{i\eta}]^T \) is the signal's state of polarisation which can be envisioned by making use of the Poincaré sphere concept [21].

However for a crossed-dipole array as illustrated in Figure 1, Equation (3) is reduced to \( \mathbf{q}(\Theta) = [E_x, E_z]^T \) consisting of only the field components induced on the horizontally and vertically polarised dipoles respectively. By assuming unity sensors sensitivities (i.e. \( V_x = V_z = 1 \)) and that the sensors and the sources are coplanar (i.e. \( \phi = 0; \Theta \triangleq [\theta, \gamma, \eta]^T \)), the manifold vector from Equation (2) thus becomes

\[
\mathbf{A}(\Theta) = \mathbf{M}(\theta) \mathcal{P}(\theta, \gamma, \eta) \in \mathbb{C}^{3N \times 1}
\]

(4)

where
Signal Model

\[
\mathcal{M} \triangleq \mathcal{M}(\theta) = [S_h(\theta), S_v(\theta)]
\]

has columns that may be viewed respectively as the spatial array manifold vector (see Equation (1)), for horizontally and vertically polarised signals arriving from the azimuth direction \( \theta \), and

\[
p \triangleq p(\Theta) = [\sin(\theta)\cos(\gamma), \sin(\gamma) e^{i\eta}]^T
\]

with \( \sin^2(\theta) \) being proportional to the power received at each sensor element with respect to the arriving angle of the signal. However in cases whereby the sensor elements are insensitive to the DOA of the incoming signal, such as the sensor arrays considered in [7, 9], \( p \) can be easily modified by setting the term \( \sin^2(\theta) = 1 \).

For some arrays that do not measure and process each polarisation component separately (as in the employment of circularly-polarised sensors in [7]), the above framework in Equation (4) can still be utilised by redefining its corresponding manifold vector as follows

\[
A(\Theta) = S(\theta, \phi) \odot q(\Theta)
\]  

(5)

where \( \odot \) is the Hadamard product, and \( q(\Theta) \) is given by

\[
q(\Theta) = V^T \cdot T(\theta, \phi) \cdot \overline{p(\gamma, \eta)}
\]  

(6)

where \( V \) is a 3 x \( N \) matrix with its \( n^{th} \) column \( [V_x^{(n)}, V_y^{(n)}, V_z^{(n)}]^T \) representing the complex voltages induced at the \( n^{th} \) sensor output in response to incoming signals with unit electric fields polarised solely in the \( x, y, \) and \( z \) directions respectively.

Note that in order to account for the mutual coupling in the array, the manifold vector described in Equations (2) and (5) may be modified to include the mutual coupling matrix \( \mathbb{C} \) [23] (i.e. \( \mathbb{C}A(\Theta) \)), which is a square complex matrix modelling the coupling effects amongst the sensor elements. Without loss of generality, and in order to simplify the notation, the mutual coupling matrix will be taken as an identity
matrix (i.e. it is assumed that there are no mutual coupling effects between the antenna elements).

**2.2 Continuous Received Signal Model**

Now let us consider an $M$-user asynchronous DS-CDMA system where the $i^{th}$ user’s transmitted baseband signal is given by

\[ m_i(t) = \sum_{n=-\infty}^{+\infty} a_i[n] c_{PN,i}(t - nT_{cs}), \quad nT_{cs} \leq t < (n + 1)T_{cs} \quad (7) \]

where \{a_i[n] \in \{-1, +1\}, \forall n \in \mathcal{N}\} is the $i^{th}$ user’s data symbol, and $T_{cs}$ is the channel symbol period. The pseudo-noise spreading waveform associated with the $i^{th}$ user, $c_{PN,i}(t)$, is modelled as

\[ c_{PN,i}(t) = \sum_{m=0}^{N_c-1} \alpha_i[m] c(t - mT_c), \quad mT_c \leq t < (m + 1)T_c \quad (8) \]

where \{\alpha_i[m] \in \{-1, +1\}, m = 0, 1, \ldots, N_c - 1\} corresponds to the $i^{th}$ user’s PN-code sequence of period $N_c = T_{cs}/T_c$, and $c(t)$ denotes the unit amplitude chip pulse-shaping waveform of duration $T_c$ which is zero outside of $0 \leq t < T_c$.

Suppose the transmitted signal due to the $i^{th}$ user arrives at the receiver via $K_i$ multipaths. To refer to the parameters $\Theta$ of the $j^{th}$ path of the $i^{th}$ user we will use two subscripts, i.e. $\Theta_{ij} = [\theta_{ij}, \gamma_{ij}, \eta_{ij}]^T$. Using the polarisation-sensitive array manifold vector $\bar{A}_{ij} \triangleq A(\theta_{ij}, \gamma_{ij}, \eta_{ij})$ of Equation (4) and letting $\beta_{ij}$, $\tau_{ij}$ be the complex path coefficient and path delay for the $j^{th}$ path of the $i^{th}$ user, the net baseband vector representation of the received signal in the presence of additive isotropic white Gaussian noise with double sided power spectral density $N_0/2$ can therefore be written as

\[ \bar{x}(t) = \sum_{i=1}^{M} \bar{A}_i \text{diag}(\beta_i) m_i(t) + n(t) \quad (9) \]
where \( n(t) \in \mathbb{C}^{2N \times 1} \) is a complex additive white Gaussian noise vector and

\[
A_i = \begin{bmatrix} A_{i1} & A_{i2} & \cdots & A_{iK_i} \end{bmatrix} \quad \in \mathbb{C}^{2N \times K_i}
\]

\[
\beta_i = \begin{bmatrix} \beta_{i1} & \beta_{i2} & \cdots & \beta_{iK_i} \end{bmatrix}^T \quad \in \mathbb{C}^{K_i \times 1}
\]

\[
m_i(t) = \begin{bmatrix} m_i(t) & m_i(t-\tau_{i1}) & \cdots & m_i(t-\tau_{iK_i}) \end{bmatrix}^T \quad \in \mathbb{R}^{K_i \times 1}
\]

By denoting \( A = [A_1, A_2, \ldots, A_M] \), \( \beta = [\beta_1^T, \beta_2^T, \ldots, \beta_M^T]^T \) and \( m(t) = [m_1^T(t), m_2^T(t), \ldots, m_M^T(t)]^T \), the above expression can be rewritten in a more compact form as

\[
x(t) = A \cdot \text{diag}(\beta) \cdot m(t) + n(t)
\]

**2.3 Discretised Received Signal Vector**

The 2\( N \)-dimensional signal vector \( x(t) \) (see point A), received from the crossed-dipole array, is then sampled at chip rate and passed through a bank of 2\( N \) tapped-delay lines (TDL), each of length \( 2N_c \) as depicted in Figure 2.

![Figure 2: Front-end receiver](image-url)
Upon concatenating its outputs, a $4N_c$-dimensional discretised signal vector (see point B) is thus formed and read for every $T_{cs}$ with the $n$th observation interval represented as

$$\bar{x}[n] = \left[ \bar{x}_1[n]^T, \bar{x}_2[n]^T, \ldots, \bar{x}_{2N}[n]^T \right]^T$$  \hspace{1cm} (11)

where $\bar{x}_k[n]$ is the $2N_c$-dimensional output frame from the $k$th TDL. Note however that due to the lack of synchronisation and with a multipath delay spread comparable to $T_{cs}$, the content of each TDL contains contributions from not only the current but also the previous and next symbols. To model such contributions, the manifold vector due to the $j$th path of the $i$th user is modified from Equation (4) to form the polar-STAR manifold vector given as

$$h_{ij} \triangleq A_{ij} \otimes \mathcal{J}^{l_{ij}} \mathcal{L}_i$$  \hspace{1cm} (12)

where $l_{ij} = \left[ \tau_{ij}/T_c \right] \bmod N_c$ is the discretised path delay; $\mathcal{L}_i$ is the $i$th user's PN-code sequence padded with $N_c$ zeros at the end, i.e.,

$$\mathcal{L}_i = \left[ \alpha_i[0], \alpha_i[1], \ldots, \alpha_i[N_c - 1], 0_{N_c}^T \right]^T$$  \hspace{1cm} (13)

and the matrix $\mathcal{J}$ (or $\mathcal{J}^T$) is a $2N_c \times 2N_c$ time down-shift (or up-shift) matrix given as follows

$$\mathcal{J} = \begin{bmatrix}
0 & 0 & \cdots & 0 & 0 \\
1 & 0 & \cdots & 0 & 0 \\
0 & 1 & \cdots & 0 & 0 \\
\vdots & \vdots & \ddots & \vdots & \vdots \\
0 & 0 & \cdots & 1 & 0
\end{bmatrix} = \begin{bmatrix}
0_{2N_c-1}^T & 0 \\
\mathbb{I}_{2N_c-1} & 0_{2N_c-1}^T
\end{bmatrix}$$  \hspace{1cm} (14)

In particular, every time the matrix $\mathcal{J}$ (or $\mathcal{J}^T$) operates on a column vector, the contents of the vector are downshifted (or upshifted) by one position, with zeros being added to the top (or bottom) of the vector, that is, $\mathcal{J}^l \mathcal{L}_i$ is a downshifted version of $\mathcal{L}_i$ by $l$ elements, and $(\mathcal{J}^T)^l$ is an upshifted version of $\mathcal{L}_i$ by $l$ elements.
For the sake of clarity, let's first consider the discretised signal vector $\mathbf{x}[n]$ associated with only the contribution of the current symbol, which may now be expressed as

$$\mathbf{x}[n] = \sum_{i=1}^{M} \mathbb{H}_i \beta_i a_i[n] + \mathbf{n}[n]$$

(15)

where $\mathbb{H}_i = [b_{i,1}, b_{i,2}, \ldots, b_{i,K_i}]$ and $\mathbf{n}[n]$ is the sampled noise vector.

By taking into account the contributions of the current, previous and next symbols, the polar-spatio-temporal discretised signal vector $\mathbf{x}[n]$ hence becomes

$$\mathbf{x}[n] = \sum_{i=1}^{M} \left[ \mathbb{H}_{i,\text{prev}} \beta_i, \mathbb{H}_i \beta_i, \mathbb{H}_{i,\text{next}} \beta_i \right] \begin{bmatrix} a_i[n-1] \\ a_i[n] \\ a_i[n+1] \end{bmatrix} + \mathbf{n}[n]$$

(16)

where $\mathbb{H}_i = [b_{i,1}, b_{i,2}, \ldots, b_{i,K_i}]$ and $\mathbf{n}[n] \in \mathbb{C}^{4N_i \times 1}$ is the sampled noise vector and

- $\mathbb{H}_{i,\text{prev}} = \left( \mathbb{I}_N \otimes (\mathbb{I}^T)^{N_i} \right) \mathbb{H}_i \in \mathbb{C}^{4N_i \times K_i}$
- $\mathbb{H}_{i,\text{next}} = \left( \mathbb{I}_N \otimes (\mathbb{I}^T)^{N_i} \right) \mathbb{H}_i \in \mathbb{C}^{4N_i \times K_i}$

Notice that by rearranging the terms in Equation (16), $\mathbf{x}[n]$ can be decoupled into four constituents, namely the desired, Inter-Symbol-Interference (ISI), Multiple-Access Interference (MAI) and noise components. Taking $D^{th}$ user as the desired user of interest, Equation (16) may thus be re-expressed as follows

$$\mathbf{x}[n] = \mathbb{H}_D \beta_D a_D[n] + I_{\text{ISI}}[n] + I_{\text{MAI}}[n] + \mathbf{n}[n]$$

(17)

where

- $\mathbb{H}_D \beta_D a_D[n]$ is the desired signal component
- $I_{\text{ISI}}[n] = \left[ \mathbb{H}_{D,\text{prev}} \beta_D, \mathbb{H}_{D,\text{next}} \beta_D \right] \begin{bmatrix} a_D[n-1] \\ a_D[n+1] \end{bmatrix}$
- $I_{\text{MAI}}[n] = \sum_{i=1, i \neq D}^{M} \left[ \mathbb{H}_{i,\text{prev}} \beta_i, \mathbb{H}_i \beta_i, \mathbb{H}_{i,\text{next}} \beta_i \right] \begin{bmatrix} a_i[n-1] \\ a_i[n] \\ a_i[n+1] \end{bmatrix}$
3 Blind Channel Estimation

This section addresses the problem of estimating the channel information for asynchronous polarisation-space-time DS-CDMA from the received TDL data blocks without recourse to the use of training signals. By performing the desired user preprocessing operation on the discretised signal vector, a music-type cost function derived in the following sub-sections, can then be applied to estimate the channel parameters of the multipaths associated with the desired user.

3.1 Operation of Preprocessor

In order to isolate the desired signal component as shown in Equation (17), we propose that the discretised signal vector $\tilde{x}[n]$ be preprocessed by the $D^{th}$ user's preprocessor which is given by

$$Z_D = I_{2N} \otimes \left( \text{diag}(F \xi_D)^{-1}F \right) \quad (18)$$

where $F$ is a $2N_c \times 2N_c$ Fourier transformation matrix, i.e.,

$$F = \left[ \Phi^0, \Phi^1, \Phi^2, \ldots, \Phi^{(2N_c-1)} \right]$$

where $\Phi = \left[ 1, \Phi^1, \Phi^2, \ldots, \Phi^{(2N_c-1)} \right]^T$

with $\Phi = \exp(-j \frac{2\pi}{2N_c})$

To see this, let's apply the preprocessor to Equation (12) as follows

$$Z_D h_{ij} = \{ I_{2N} \otimes \left( \text{diag}(F \xi_D)^{-1}F \right) \} \cdot (A_{ij} \otimes I_{i,j} \xi_i)$$

$$= A_{ij} \otimes \{ (\text{diag}(F \xi_D)^{-1}F).(I_{i,j} \xi_i) \}$$

$$= A_{ij} \otimes \{ \text{diag}(F \xi_D)^{-1} \text{diag}(F \xi_j) \Phi^{ij} \} \quad (20)$$

Notice that the above expression can be reduced to simply $A_{Dj} \otimes \Phi^{(\nu)}$ if and only if $i = D$ which corresponds to the desired user of interest. Hence by applying the same operation to Equation (17), the discretised signal vector is thus transformed to

$$\tilde{y}[n] = Z_D \tilde{x}[n]$$

$$= \mathbb{H}_D \beta_D a_D[n] + Z_D l_{\text{SI}}[n] + Z_D l_{\text{MAI}}[n] + Z_D n[n] \quad (21)$$

where $\mathbb{H}_D = [A_{D1} \otimes \Phi^{\nu_1}, A_{D2} \otimes \Phi^{\nu_2}, \ldots, A_{DK_D} \otimes \Phi^{(\nu_K_D)}]$. 
3.2 Spatial-Temporal Smoothing Technique

However the second order statistics of the vector $y[n]$, instead of providing a basis for the desired signal subspace, would result in a rank deficiency with the desired signal subspace dimension being reduced to one. This is not due to the signal coherence problem (signal clustering or diffused signals) but because, as shown in Equation (21), the columns of $\mathbf{H}_D$ are linearly combined by the path coefficient vector $\mathbf{\beta}_D$, hence its contribution to the observation space of $y[n]$ will consequently lead to a subspace of only one dimension. To restore the dimensionality of this subspace back to $K_D$, we will make use of the Vandermonde structure of the submatrices of $\mathbf{H}_D$ provided by the preprocessing operation. This can be achieved by performing a technique referred to as temporal smoothing, a concept similar to that of spatial smoothing described in [22].

![Figure 3: Temporal smoothing procedure.](image)

For clarity, let's reshape the $y[n]$ vector to a $C^{4N_e \times 1}$ vector to $\mathbf{Y}[n] \in C^{2N \times 2N_e}$ matrix so that each successive two rows reflect the preprocessed output for each crossed-dipole sensor, as illustrated in Figure 3. By extracting a set of $Q$ (where $Q = 2N_e - d + 1$) overlapping submatrices of length $d$ (where $d < 2N_e$), and concatenating each of the submatrices via vectorisation, $Q$ concatenated subvectors
each of length $Nd$ are thus formed, i.e.

$$y_q[n] = \text{vec}(Y_q[n]), \quad \forall q = 1, 2, \ldots, Q \quad (22)$$

where $\text{vec}(\bullet)$ is the row-wise vectorisation operator. With that, the $2Nd \times 2Nd$ temporal-smoothed covariance matrix $R_{\text{smooth}}$ can therefore be obtained as follows

$$R_{\text{smooth}} = \frac{1}{Q} \sum_{q=1}^{Q} R_{y_q} \quad (23)$$

where $R_{y_q}$ is the covariance matrix obtained from the subvector $y_q[n]$. Note that the dimension can only be successfully restored to $K_D$, provided that $Q \geq K_D$. This technique also applies for paths arriving from the same direction (co-directional). However, for paths arriving at the same time (co-delay), singularity in $R_{\text{smooth}}$ will occur. This special case cannot be resolved for a general array geometry but for a uniform linear array where spatial smoothing can be performed on top of Equation (23) to form the spatial-temporal-smoothed covariance matrix $R_{\text{STsmooth}}$.

### 3.3 Polarisation-Angle-Delay Estimation (PADE)

From the above discussion, it can be shown that by utilising the polar-STAR manifold vector, the MUSIC-type cost function is based on the following criterion:

$$\xi_1(\theta, l, \gamma, \eta) = (A(\Theta) \otimes \Phi_l)H E_n \Sigma_n H (A(\Theta) \otimes \Phi_l) = p(\Theta)H (M(\theta) \otimes \Phi_l)H E_n \Sigma_n H (M(\theta) \otimes \Phi_l) p(\Theta) \quad (24)$$

where $\Phi_l$ is a subvector of $\Phi$ with length $d$, and $E_n$ is a matrix whose columns are the generalised noise eigenvectors of $(R_{\text{STsmooth}}, D)$ due to the transformed noise in Equation (21), with $D$ representing the spatial-temporal smoothed diagonal matrix $Z_D Z_D^H$. However, such operation involves a multidimensional search over $\theta$, $l$, $\gamma$ and $\eta$ for its minima. A more efficient minimisation method can be obtained by letting the 2 x 2 matrix $B \triangleq \begin{bmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{bmatrix}$ be

$$B \triangleq B(\theta, l) = (M(\theta) \otimes \Phi_l)H E_n \Sigma_n H (M(\theta) \otimes \Phi_l) \quad (25)$$

Note that minimisation over the polarisation space of $p(\Theta)$ is equivalent to finding the
eigenvectors corresponding to the minimum eigenvalues of $\mathbb{B}$. Hence by dropping the two-dimensional vector $\mathbf{p}(\Theta)$ and applying the quadratic formula, it can be established that Equation (24) can be simplified to the following 'STAR-Subspace' cost function

$$\xi_2(\theta, l) = \text{trace}(\mathbb{B}(\theta, l)) - \sqrt{\text{trace}(\mathbb{B}(\theta, l))^2 - 4\text{det}(\mathbb{B}(\theta, l))}$$

(26)

where $\text{trace}(\bullet)$ denotes the trace operation and $\text{det}(\bullet)$ represents the determinant of the $2 \times 2$ matrix. Now let $\xi_2^{\text{min}}$ be the minima obtained from the spectrum constructed using the cost function $\xi_2(\theta, l)$ in Equation (26). The location of the minima, as such, provides the joint estimate of its direction of arrival (DOA) and time of arrival (TOA). Its corresponding polarisation parameters, on the other hand, are estimated as follows:

$$\hat{\gamma} = \tan^{-1}(|\rho|)$$

(27a)

$$\hat{\eta} = \arg\left(\frac{\rho}{|\rho|}\right)$$

(27b)

where

$$\rho = \begin{cases} 
\frac{(\xi_2^{\text{min}} - 2b_{11})\sin(\hat{\theta})}{2b_{12}} \\
2b_{21}\sin(\hat{\theta})/(\xi_2^{\text{min}} - 2b_{22})
\end{cases}$$

(27c)

It is worthwhile noting that the above expression $\rho$ in Equation (27c), if necessary, can be further simplified in most circumstances since the value $\xi_2^{\text{min}}$ is usually close to zero.

Consider an ill-conditioned situation in which two or more multipaths of differing polarisation are closely located in space and time, such that their minima are indistinguishable or unresolvable from one another in the space and time domain. In such an event, the eigenvalues of the matrix $\mathbb{B}$ would be approximately near to zero, thus degenerating $\mathbb{B}$ to a near trivial matrix. Hence, by exploiting the supplementary eigenvalue of the matrix $\mathbb{B}$, the following supplementary 'STAR-Subspace' cost function can be utilised to detect such occurrences.

$$\xi_3(\theta, l) = \text{trace}(\mathbb{B}(\theta, l)) + \sqrt{\text{trace}(\mathbb{B}(\theta, l))^2 - 4\text{det}(\mathbb{B}(\theta, l))}$$

(28)

The complete procedure outlining the major steps of the proposed polarisation-angle-delay channel estimation algorithm is summarized as shown below:
1. Sample the crossed-dipole array output at chip rate and collect the data by concatenating the tapped-delay lines (TDL) contents to form the discretised signal vector of Equation (11).

2. Apply the desired user's preprocessor (Equation (18)) onto the discretised signal vector and perform the temporal smoothing technique to restore the dimensionality of the desired signal subspace. Form the matrix $\mathbf{R}_{\text{smooth}}$. Note that for paths arriving at the same time (co-delay), spatial smoothing (only for linear arrays) can be performed on top of the temporal smoothing technique to form the spatial-temporal-smoothed covariance matrix.

4. Apply the cost function in Equation (26), using the generalised eigenvector decomposition of $\mathbf{R}_{\text{smooth}}$, to obtain the joint estimate of the direction of arrival (DOA) and time of arrival (TOA) of the desired user's multipaths. The corresponding polarisation parameters are estimated using the set of analytical equations in Equation (27).

5. In an ill-conditioned situation in which two or more multipaths of differing polarisation are closely located in space and time, the supplementary cost function in Equation (28) can be employed to detect such occurrences.
4 Simulation Studies

In this section, the effectiveness of the PADE algorithm employed for the crossed-dipole array is investigated by means of a set of representative computer simulations. The performance of the algorithm will first be studied, followed by its capability in resolving closely-located paths and finally in an ill-conditioned situation whereby the space and time dimensions are no longer differentiable.

4.1 Performance of PADE algorithm

In this section, several illustrative examples are presented to demonstrate the key features of the proposed PADE algorithm. Consider a linear array consisting of $N = 5$ crossed-dipoles (with each having one half-wavelength spacing) operating in the presence of $M = 3$ co-channel CDMA users, where each user is being assigned an unique Gold sequence of length $N_c = 31$. The array is assumed to collect 200 data symbols for processing.

<table>
<thead>
<tr>
<th>Path</th>
<th>$\theta_{ij}$</th>
<th>$l_{ij}$</th>
<th>$\gamma_{ij}$</th>
<th>$\eta_{ij}$</th>
<th>$\theta_{kj}$</th>
<th>$l_{kj}$</th>
<th>$\gamma_{kj}$</th>
<th>$\eta_{kj}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$j = 1$</td>
<td>40°</td>
<td>$8T_c$</td>
<td>0°</td>
<td>-</td>
<td>50°</td>
<td>$18T_c$</td>
<td>80°</td>
<td>30°</td>
</tr>
<tr>
<td>$j = 2$</td>
<td>50°</td>
<td>$20T_c$</td>
<td>90°</td>
<td>-</td>
<td>70°</td>
<td>$10T_c$</td>
<td>10°</td>
<td>-70°</td>
</tr>
<tr>
<td>$j = 3$</td>
<td>60°</td>
<td>$15T_c$</td>
<td>45°</td>
<td>90°</td>
<td>80°</td>
<td>$5T_c$</td>
<td>40°</td>
<td>120°</td>
</tr>
<tr>
<td>$j = 4$</td>
<td>85°</td>
<td>$10T_c$</td>
<td>45°</td>
<td>-90°</td>
<td>95°</td>
<td>$20T_c$</td>
<td>50°</td>
<td>90°</td>
</tr>
<tr>
<td>$j = 5$</td>
<td>100°</td>
<td>$5T_c$</td>
<td>30°</td>
<td>-10°</td>
<td>95.5°</td>
<td>$20T_c$</td>
<td>45°</td>
<td>0°</td>
</tr>
<tr>
<td>$j = 6$</td>
<td>100°</td>
<td>$21T_c$</td>
<td>5°</td>
<td>-180°</td>
<td>110°</td>
<td>$6T_c$</td>
<td>70°</td>
<td>100°</td>
</tr>
<tr>
<td>$j = 7$</td>
<td>120°</td>
<td>$15T_c$</td>
<td>20°</td>
<td>80°</td>
<td>130°</td>
<td>$25T_c$</td>
<td>30°</td>
<td>-160°</td>
</tr>
<tr>
<td>$j = 8$</td>
<td>130°</td>
<td>$25T_c$</td>
<td>60°</td>
<td>120°</td>
<td>140°</td>
<td>$15T_c$</td>
<td>20°</td>
<td>5°</td>
</tr>
</tbody>
</table>

Assume that user 1 is the desired user, with an input SNR of 20dB, together with 2 other interferers each constituting a SIR of -20dB (i.e. near-far problem). All 3 users are assumed to have 8 multipath rays each, with their parameters as listed in Table I. By partitioning the array into 2 overlapping subarrays (each having 4 crossed-
dipoles) for spatial smoothing and setting $d = 50$ for temporal smoothing, it is seen from Figure 4 that all the 8 multipaths, as well as the co-delay and co-directional paths, can be identified/estimated successfully using the proposed algorithm. Their polarisation parameters can also be estimated as shown in Table II by using the set of analytical equations given in Equation (27). Notice that the number of resolvable paths is now no longer limited by the number of sensors available in the array.

![Figure 4: MUSIC-type spectrum of desired user.](image)

<table>
<thead>
<tr>
<th>Path</th>
<th>$j=1$</th>
<th>$j=2$</th>
<th>$j=3$</th>
<th>$j=4$</th>
<th>$j=5$</th>
<th>$j=6$</th>
<th>$j=7$</th>
<th>$j=8$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\hat{\eta}_{1j}$</td>
<td>0.06°</td>
<td>89.89°</td>
<td>45.03°</td>
<td>44.99°</td>
<td>29.91°</td>
<td>5.06°</td>
<td>19.91°</td>
<td>59.80°</td>
</tr>
<tr>
<td>$\hat{\eta}_{2j}$</td>
<td>-</td>
<td>-</td>
<td>89.91°</td>
<td>-90.09°</td>
<td>-10.08°</td>
<td>-179.25°</td>
<td>79.90°</td>
<td>120.02°</td>
</tr>
</tbody>
</table>

Unlike linearly polarised array, it is also clear that the crossed-dipole array is capable of handling any arbitrary polarised paths, including signal path which is horizontally polarised (path 1), vertically polarised (path 2), left-hand circularly polarised (path 3) or right-hand circularly polarised (path 4). The performance of the estimation is shown to be relatively consistent irregardless of the received signal's polarisation [17].
Figure 5 depicts the standard deviations of the polarisation, DOA and TOA estimates of the desired user for a normal-incidence desired signal path delayed over $15T_c$ with a SNR of -10dB. It is assumed that the desired user operates in the presence of 4 other interferers (with each interferer having 8 multipaths) individually constituting a desired signal to interference ratio of 40dB. The results are averaged over 200 Monte-Carlo simulations. Although different, the three parameters (deviations) of the desired signal are shown for convenience on the same graph and are plotted against the desired signal's polarisation parameters $\gamma$ (with $\eta = 0$) and $\eta$ (with $\gamma = \pi/4$), to demonstrate the performance in response to linear (ranging between horizontal to vertical polarisations) and elliptical (ranging between right-hand circular to left-hand circular polarisations) polarisations respectively. Note that the standard deviation of the polarisation estimate is defined as the angular distance $\delta$ on the Poincaré sphere, where $0 \leq \delta \leq \pi$, obtained using

$$\cos(\delta) = \cos(2\gamma) \cos(2\hat{\gamma}) + \sin(2\gamma) \sin(2\hat{\gamma}) \cos(\eta - \hat{\eta}) \quad (29)$$

From Figure 5, it is therefore apparent that in a poor SNR environment, with strong interferers constituting a near-far situation, the proposed algorithm is still able to provide relatively consistent and accurate estimates irrespective of the polarisations of the received signal.
Next, let's take a closer look at the effect of polarisation on the performance of a crossed-dipole array as compared with a vertically polarised array and a horizontally polarised array. Consider first the case of a single signal path with $\theta = 80^\circ$ and $l = 22T_c$. Assuming an SNR of 0dB and SIR = -30dB, the standard deviation of the estimates with respect to its orientation angle $\kappa$ (where $\tan(2\kappa) = \tan(2\gamma) \cdot \cos(\eta)$) and ellipticity angle $\epsilon$ (where $\sin(2\epsilon) = \sin(2\gamma) \cdot \sin(\eta)$) are averaged over 100 Monte Carlo runs and plotted as shown in Figure 6. As expected, the estimates deteriorate rapidly as the signal tends to vertical polarisation ($\kappa \to 90^\circ$) for a horizontally polarised array; as well as when the signal tends to horizontal polarisation ($\kappa \to 0^\circ$ or $180^\circ$) for a vertically polarised array as illustrated in Figure 6a. Similarly from Figure 6b, the estimates degenerate as its elliptical polarisation approaches linear ($\epsilon \to 0^\circ$) for a vertically polarised array. In contrast, the estimates due to a crossed-dipole array have in general a lower and relatively constant standard deviation irregardless of the received signal's polarisation. But note that this deviation will increase as the signal gravitates towards the endfire direction of the array, especially when its polarisation tends horizontal.
Figure 6: Standard deviation of estimates versus (a) orientation angle $\kappa$ with $\epsilon = 0^\circ$ (linear polarisation) (b) ellipticity angle $\epsilon$ with $\kappa = 0^\circ$ (elliptical polarisation), using crossed-dipole array (solid line), horizontally-polarised array (dashed line) and vertically-polarised array (dotted line).

### 4.2 Studies of closely-located paths

Let's compare the performance of a crossed-dipole array with that of an equivalent polarisation-insensitive array, with the latter assumed to be omitting any polarisation mismatches. For simplicity, we will assume user 1 to have only 2 linearly polarised multipaths: $(\theta_{11}, l_{11}, \gamma_{11}, \eta_{11}) = (80^\circ, 10T_c, 20^\circ, 0^\circ)$ and $(\theta_{12}, l_{12}, \gamma_{12}, \eta_{12}) = (82^\circ, 11T_c, \gamma, 0^\circ)$ with the polarisation difference defined as
\( \Delta \gamma = (\gamma - 20^\circ) \). Using the same scenario as that in Table I, the standard deviation of the estimates associated with the first multipath is plotted as shown in Figure 7.

![Figure 7](image-url)

**Figure 7**: Standard deviation of (a) time of arrival (TOA) and (b) direction of arrival (DOA) estimates for the first multipath of the desired user, in the presence of a second multipath, versus the SNR.

As expected, paths that are well separated in polarisation exhibit a better standard deviation, with those due to the crossed-dipole array having in general a lower deviation (even when the polarisation difference \( \Delta \gamma = 0 \)) than that of the polarisation-insensitive array. Such deviation is also observed to be deteriorating with decreasing angular/temporal separation. Take the case whereby user 1 has the following two identically polarised (\( \gamma = 20^\circ \) and \( \eta = 0^\circ \)) signal paths: 

(\( \theta_{11}, l_{11} \)) = \((80^\circ, 10T_c)\) and (\( \theta_{12}, l_{12} \)) = \((80^\circ + \Delta \theta, 11T_c)\) with a SNR = 5dB. The standard deviation of the estimates due to the first signal path is plotted versus their angular separation \( \Delta \theta \) as illustrated in Figure 8a. Similarly, Figure 8b depicts its response towards their temporal separation \( \Delta l \), with the second path given by (\( \theta_{12}, l_{12} \)) = \((81^\circ, 10T_c + \Delta l)\). Again the crossed-dipole array provides better estimation as compared with that of the polarisation-insensitive array where both estimates degenerate, as anticipated, with decreasing angular and temporal separations.
Hence it is apparent that by incorporating the polarisation dependent processing, the crossed-dipole array can improve the accuracy and resolution of the estimates considerably, especially when the paths are well separated in polarisation. This improvement is even more significant for paths in a poor SNR environment and paths that are very closely located. To get a clearer picture, let's take a closer look at the effect of polarisation on the spectrum due to the crossed-dipole array as compared with that of the polarisation-insensitive array. Now consider the case of a vertically polarised signal path of \((\theta_{11}, l_{11}, \gamma_{11}, \eta_{11}) = (50^\circ, 5T_c, 90^\circ, 0^\circ)\) together with a closely-located linearly polarised multipath given by

**Figure 8:** Standard deviation of time of arrival (TOA) and direction of arrival (DOA) estimates for one of the two multipaths of the desired user, versus (a) DOA separation and (b) TOA separation.
\((\theta_{12}, l_{12}, \gamma_{12}, \eta_{12}) = (51^\circ, 6T_c, 30^\circ, 0^\circ)\). Assuming an SNR = -3dB and SIR = -30dB, it is seen from Figure 9a that the crossed-dipole array is able to clearly resolve the two signal paths; whereas that of the polarisation-insensitive array has their peaks collapsed together as depicted in Figure 9b.

4.3 Example of ill-conditioned event

Finally, consider an ill-conditioned case whereby 2 paths are so closely located that even the crossed-dipole array fails to distinguish/resolve their spatial and temporal locations. Let's take user 2 as the desired user, with its 4th and 5th paths being so closely located such that the resolution of their peaks collapse and merge into one as illustrated in Figure 10a. By making use of the supplementary cost function in Equation (28), such an occurrence can be detected as depicted in Figure 10b. The capability and proximity limit of such detection is found to be even better if the
polarisation difference between the paths is higher or the strength of the paths increases.

Figure 10: (a) Spectrum of user 2 with point A indicating the ill-conditioned event due to the unresolvable 4th and 5th paths, (b) Supplementary spectrum of user 2 with point B indicating the detected ill-conditioned occurrence.
5 Conclusion

A summary of the work is presented in this section, followed by the possible future directions of the research.

5.1 Summary

In this report an efficient near-far resistant channel estimation algorithm is presented using a crossed-dipole array in an asynchronous completely polarised multipath DS-CDMA system. The proposed approach is based on the polar-STAR manifold vector parameterisation and the transformation of the data to another domain. However, in this parameterisation the polar-STAR manifold vectors of the various paths of the desired user are linearly combined by the fading coefficient. This leads to the collapse of the desired signal subspace (which is a subset of the overall signal subspace) to a rank-1 subspace. To overcome this problem and to restore the rank of the desired signal subspace a temporal smoothing technique is employed operating on the transformed domain. This leads to the 'STAR-Subspace' cost function of Equation (26) which is used in conjunction with the analytical expressions of Equation (27) to provide a joint estimation of the polarisation-angle-delay parameters associated with the desired user.

In contrast to the linearly polarised arrays, the crossed-dipole arrays employed in this report are able to capture the polarisation diversity in the signal, hence providing a better detection and estimation performance regardless of its received polarisation. By exploiting the inherent polarisation information in the signal, the joint space-time channel estimator is also able to resolve closely-located paths that otherwise cannot be resolved using its equivalent polarisation-insensitive array. In the event when the paths are so closely-located, such that the proposed algorithm fails, its supplementary 'STAR-Subspace' cost function of Equation (28) can be employed to detect such an occurrence. Finally it is important to point out the number of multipaths that can be resolved by the proposed algorithm is not constrained by the
number of sensors available in the array. This is due to the way of collecting the received data (see Equation (16)) and the extended dimensionality of the polar-STAR array manifold vector of Equation (12).

5.2 Directions of Future Research

5.2.1 Applications of 'STAR-Subspace' function

The 'STAR-Subspace' cost function as shown in Equation (26) is not specifically derived only for polarisation-angle-delay estimation. It can also be applied for other applications as long as there exists no more than four unknown parameters in the expression as depicted in Equation (24), with two of the parameters jointly estimated. For more than four unknown parameters, subspace-fitting techniques would deem to be more appropriate as illustrated in [24]. The application of the 'STAR-Subspace' cost function had been successfully applied in [25] in the development of a blind space-time receiver based on multiple point or space-diffused signal paths in an asynchronous DS-CDMA system. The direction-of-arrivals and time-of-arrivals of the desired user's multipaths are jointly estimated with the weighted spread factor found using a set of analytical expressions. The receiver is robust to erroneous or incomplete channel estimation since its operation requires only the existence of an estimable path due to the desired user separable/identifiable in at least one of the following domains: space, time and code.

5.2.2 Space-Time Diffused Receiver

Having investigated the possibility of employing the 'STAR-Subspace' cost function in space-diffused channel, it would also be interesting to look at subspace-based space-time receiver that can cope in a space-time diffused environment (as proposed in the EPSRC project, "Integrated Multiuser Reception and Vector-Channel Estimation"). The aim is to develop a blind/semi-blind receiver based on a superresolution space-time channel estimator. Likewise, the estimator would provide
a joint estimates of the direction-of-arrivals and time-of-arrivals associated with the desired user's multipaths. However, instead of estimating only the angular spread as in the case for the above-mentioned receiver, the temporal spread might have to be estimated as well. The space-time diffused single/multiuser receiver will also be near-far resistant, with the ability of mitigating the effects of ISI and MAI interferences.

### 5.2.3 Blind Polar-Space-Time Receiver

Following the investigation of the polarisation-angle-delay estimation as described in this report, the work would be complete if a receiver could be developed based on the estimated multipath's channel parameters. Emphasis would be placed on the blind or semi-blind subspace-type polar-space-time receiver. The idea is to incorporate the channel estimator into the receiver so as to combine the desired multipath rays and, at the same time, to provide suppression/cancellation of the ISI and MAI interferences. And since the receiver is subspace-based, it will also be able to perform under near-far condition environment. But unlike conventional polarisation-insensitive sensors, the receiver should be able to receive signals of any arbitrary state of polarisation, and with the capability of resolving closely-located paths.
References


