MODELLING and ESTIMATION of MUTUAL COUPLING BETWEEN ARRAY ELEMENTS

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ABSTRACT
The adverse effect of mutual coupling between the elements of an array, on the performance of super-resolution techniques is demonstrated. An analytical modelling of the mutual coupling is presented, and its effects are modelled in the form of a complex square matrix, the Mutual Coupling Matrix (MCM). The MCM depends on the array geometry and the array electrical characteristics, but not on the direction of the incoming signals. Based on the modelling of the mutual coupling effects, a new technique is proposed for estimating the Mutual Coupling Matrix of an array of general geometry. The proposed method employs one pilot source, and uses an extra element at some distance away from the original array.

1. INTRODUCTION
Signal-subspace techniques ignore the mutual coupling between the array elements, since they assume that every element is isolated from the other elements. Furthermore, these techniques break down in the presence of unaccounted mutual coupling between the array elements. For instance, in Figure 1 the spectrum of the MuSiC algorithm is illustrated for a planar array with 7 elements operating in the presence of two sources at (30°, 0°) and (65°, 0°). In this figure, the solid line corresponds to the case where the mutual coupling effects are known, while the dotted line corresponds to the case where there are uncertainties associated with the mutual coupling. It is apparent from Figure 1, that in the presence of unaccounted mutual coupling effects, the MuSiC algorithm fails to identify, correctly, the true signal environment. Thus, mutual coupling uncertainties affect significantly the performance of signal-subspace techniques, especially in the case where the interelement spacing is small.

Previous attempts to address the mutual coupling uncertainties problem include techniques:
- based on measurements of the impedances of the array elements, [1, 2],
- applied on particular array geometries only, [3],
- that reduce rather than eliminate the mutual coupling uncertainties, [4, 5].

In Section 2, the mathematical modelling of the mutual coupling uncertainties will be described in the form of a complex square matrix referred to as the Mutual Coupling Matrix. In Section 3, an alternative modelling of the data covariance matrix will be presented. Then, in Section 4, the algorithm proposed for estimating the mutual coupling matrix will be described in detail. Finally, in Section 5 the paper will be concluded.

2. MATHEMATICAL MODEL
Consider an array of N elements with mutual coupling uncertainties, that operates in the presence of one pilot source with power normalized to unity. Signal-Subspace techniques are based on the knowledge
of the 2nd-order statistics of the received signals, i.e. the data covariance matrix, $\mathbf{R}_{xx}$. In the absence of any mutual coupling effects the matrix $\mathbf{R}_{xx}$ can be represented as follows:

$$\mathbf{R}_{xx} = \mathbf{S} \cdot \mathbf{S}^{H} + \sigma^{2} \cdot \mathbb{I} \quad (1)$$

where $\mathbf{S}$ is the Source Position Vector (SPV) or Manifold Vector, $\sigma^{2}$ is the power of thermal plus isotropic noise and $\mathbb{I}$ is the identity matrix.

However, in the presence of mutual coupling uncertainties, Equation 1 is not anymore an accurate representation of the data covariance matrix. A model of the mutual coupling uncertainties and the way they are taken into account by the data covariance matrix will be described next. Consider the pilot signal at the elements of the array. In the presence of mutual coupling, a part of the power received by the elements will be absorbed, while the rest of the power will be re-radiated. Although the original signal may come from a far-field source, which implies plane-wave propagation, the mutual coupling effects between the array elements involve spherical wave propagation. As a consequence of the reciprocity theorem, the electrical characteristics of the array elements remain the same, irrespectively of the elements acting either as receivers, or as transmitters. In addition to the direct signal, every element receives also signals which are re-radiated by other elements. The amplitude of these signals depends on the power of the re-radiated signal. Furthermore, every element may introduce a random phase to its re-radiating signal, while the propagation path will introduce an additional electrical phase associated with the propagation delay from one element to another. Finally, the electrical characteristics of the elements in the direction of the propagation path, as well as, the propagation losses should be taken into account. Consequently, the data covariance matrix in the presence of mutual coupling should be modelled as follows:

$$\mathbf{R}_{xx} = \mathbf{C} \cdot \mathbf{S} \cdot \mathbf{S}^{H} \cdot \mathbf{C}^{H} + \sigma^{2} \cdot \mathbb{I} \quad (2)$$

where $\mathbf{C}$ is a matrix referred to as Mutual Coupling Matrix (MCM). It can be shown that the MCM $\mathbf{C}$ can be expressed as:

$$\mathbf{C} = \mathbf{A} \odot \mathbb{L} \odot e^{\mathbf{\Phi}} \odot \mathbf{G} \odot e^{j\mathbb{D}} \quad (3)$$

where $\odot$ denotes Hadamard product and:

$$\mathbb{L} = \begin{bmatrix} 1 & \cdots & l_{1N} \\ \vdots & \ddots & \vdots \\ l_{N1} & \cdots & 1 \end{bmatrix}, \quad \mathbf{\Phi} = \begin{bmatrix} 0 & \phi_{2} & \cdots & \phi_{N} \\ \phi_{1} & 0 & \cdots & \phi_{N} \\ \vdots & \ddots & \vdots & \vdots \\ \phi_{1} & \phi_{2} & \cdots & 0 \end{bmatrix},$$

$$\mathbf{A} = \begin{bmatrix} \sqrt{1 - \alpha_{1}} & \cdots & \sqrt{\alpha_{N}} \\ \vdots & \ddots & \vdots \\ \sqrt{1 - \alpha_{1}} & \cdots & \sqrt{1 - \alpha_{N}} \end{bmatrix} \quad \mathbf{G} = \begin{bmatrix} 1 & \cdots & g_{1N} \\ \vdots & \ddots & \vdots \\ g_{N1} & \cdots & 1 \end{bmatrix} \quad \mathbb{D} = \begin{bmatrix} 0 & \cdots & d_{1N} \\ \vdots & \ddots & \vdots \\ d_{N1} & \cdots & 0 \end{bmatrix}.$$
where $\mathbf{S}$ is the Source Position Vector of the pilot source and $\mathbf{C}$ is the MCM for the array. Every element that re-radiates the received signal due to the pilot source, can be seen as a source that is fully correlated with the pilot source. Thus, there are $N+1$ sources present in the environment of an array of $N$ elements. In order to provide a unique solution, the number of elements should be, at least, equal to the number of sources. This can be achieved by reducing the number of re-radiating signals by one, which is equivalent to the assumption that one element does not re-radiate the received signal and thus, it does not contribute to mutual coupling. Alternatively, the number of elements may be increased by one, non-re-radiating, element. This can be achieved by using an extra element at some distance from the rest of the elements, so that, by virtue of the distance, its mutual coupling contribution to the elements of the original array will be negligible.

Any directional element can be seen as an isotropic element that is, subsequently, masked by the directional electrical characteristics of the original element. Thus, since the directional characteristics and the power gain of the elements are already taken into account by matrices $\mathbf{G}$ and $\mathbf{I}$, it may be assumed that every element re-radiates the same percentage of received power to the rest of the elements. In this case, it can be shown that matrix $\mathbf{R}$ can also be modelled as:

$$\mathbf{R} = \mathbf{S} \mathbf{S}_r \cdot \mathbf{e}^{\mathbf{H}} \cdot \left[ \mathbf{S} \mathbf{S}_r \right]^H$$

(6)

with

$$\mathbf{c} = \begin{bmatrix} \sqrt{1 - \alpha} \\ \frac{1}{k} \odot \mathbf{S} \end{bmatrix} \text{ and } \mathbf{k} = \sqrt{\alpha} \cdot \mathbf{e}^{j\theta}.$$

In Equation 6, matrix $\mathbf{S}_r$ represents a matrix with columns the SPVs of the re-radiated signals, i.e:

$$\mathbf{S}_r = \left[ \mathbf{S}_1 \mathbf{S}_2 \cdots \mathbf{S}_N \right].$$

(7)

These SPVs take into account spherical-wave propagation. The elements of the $i^{th}$ column of $\mathbf{S}_r$ are described by the following equation:

$$S_i(k) = \begin{cases} 0, & k = i \\ \frac{\alpha}{2\pi d_k} e^{j \kappa d_k}, & k \neq i
\end{cases}, \; i,k = 1 \cdots N.$$  

(8)

Using the definitions in Section 2, it can be proved that the matrix $\mathbf{S}_r$ can be expressed as follows:

$$\mathbf{S}_r = \mathbf{I} \odot \mathbf{e}^{j\mathbf{H}} - \mathbf{I}.$$  

(9)

Furthermore, it can be shown that:

$$\mathbf{1} \cdot \mathbf{k}^T - \text{diag}(\mathbf{1} \cdot \mathbf{k}^T)) + \mathbf{I} = \mathbf{A} \odot \mathbf{e}^{\mathbf{H}}.$$  

(10)

Using Equations 9 and 10, in conjunction with the model of the MCM $\mathbf{C}$ described in Equation 3, it is:

$$\mathbf{C} = (\mathbf{1} \cdot \mathbf{k}^T - \text{diag}(\mathbf{1} \cdot \mathbf{k}^T)) + \mathbf{I} \odot \mathbf{G} \odot (\mathbf{S}_r + \mathbf{I}).$$  

(11)

When the array geometry and the directional electrical characteristics of the elements are known, then matrices $\mathbf{G}$ and $\mathbf{S}_r$ are also known. Thus, the only unknown in Equation 11 for the estimation of the MCM $\mathbf{C}$ is vector $\mathbf{k}$. Note, that the vector $\mathbf{k}$, as well as, the scalar $\alpha$ can be estimated as follows:

$$\begin{bmatrix} \sqrt{1 - \alpha} \\ \mathbf{k} \end{bmatrix} = \mathbf{c} \odot \begin{bmatrix} 1 \\ \mathbf{1} \odot \mathbf{S} \end{bmatrix}.$$  

(12)

Note that $\odot$ denotes element-by-element division. By pre- and post-processing the matrix $\mathbf{R}$ in Equation 6 with the matrix $[\mathbf{S} \mathbf{S}_r]^{-1}$ it is possible to isolate the product $\mathbf{c} \cdot \mathbf{c}^H$ and then to estimate uniquely vector $\mathbf{c}$.

Using the above ideas, a new algorithm was developed to estimate the MCM $\mathbf{C}$. This algorithm is proposed in the following section in a step form.

4. MUTUAL COUPLING ESTIMATION ALGORITHM

Consider one pilot source at some direction $(\theta, \phi)$, operating in the presence of an array of $N$ elements.

**Initialization**

**step 1**: Place an extra element in the environment of the array at some known location and at some distance away from the original array. A new array of $N+1$ elements is formed.

**step 2**: Evaluate the Source Position Vector $\mathbf{S}$ which corresponds to the new array and the pilot source located at $(\theta, \phi)$. Evaluate the columns of matrix $\mathbf{S}_r$ for all the elements of the original array: $\mathbf{S}_1$, $\mathbf{S}_2$, $\cdots$, $\mathbf{S}_{N-1}$ and $\mathbf{S}_N$. Form the matrix $\mathbf{S}$:

$$\mathbf{S} = \left[ \mathbf{S} \mathbf{S}_1 \cdots \mathbf{S}_N \right].$$

**step 3**: Measure the data covariance matrix $\mathbf{R}_{ee}$ due to the pilot source. Eliminate the noise effects on $\mathbf{R}_{ee}$ by subtracting the noise co-variance matrix. Thus, matrix $\mathbf{R}$ is formed.

**Mutual Coupling Estimation**

**step 4**: Estimate matrix $\mathbf{R}_1$:

$$\mathbf{R}_1 = \mathbf{S}^{-1} \cdot \mathbf{R} \cdot \mathbf{S}^{-H}.$$  

The first column of matrix $\mathbf{R}_1$, divided by the square root of its first element, is vector $\mathbf{c}$.  

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step 5: Repeat step 2 for the original array only. Thus, vectors \( S_1, S_2, \ldots, S_{N-1} \) and \( S_N \) are defined. Form matrix \( S_r \):

\[
S_r = [S_1 \ S_2 \ \cdots \ S_{N-1}]
\]

and matrix \( G \) from the electrical characteristics of the elements of the original array.

step 6: Find scalar \( a \) and vector \( b \) as follows:

\[
\begin{bmatrix}
\sqrt{1 - a} \\
b
\end{bmatrix} = \mathcal{C} \begin{bmatrix}
1 \\
1 \otimes S
\end{bmatrix}
\]

step 7: Calculate the Mutual Coupling Matrix \( \mathcal{C} \):

\[
\mathcal{C} = (1 \cdot b^T - \text{diag}(\text{diag}(1 \cdot b^T)) + 1) \otimes G \otimes (S_r + 1).
\]

In order to demonstrate the performance of the suggested method, consider a 7-element array operating in the presence of two sources at \((35^\circ, 0^\circ)\) and \((65^\circ, 0^\circ)\). Figure 2 shows the spatial configuration of the array. The extra element is also shown in Figure 2, at the coordinates \((10,10)\). Figure 3 shows the spectrums of the MuSiC cost function, obtained using the above array. The solid line is obtained using the MCM estimated with the proposed algorithm, and a pilot source at \((50^\circ, 0^\circ)\). Furthermore, the dotted line corresponds to the MuSiC spectrum, obtained in the presence of mutual coupling uncertainties. Finally, in order to demonstrate the effect of the distance of the extra element from the original array, the spectrum of the MuSiC cost function for varying distances of the extra element is shown in Figure 4. It is apparent from Figure 4, that the greater the distance of the extra element from the array is, the deeper the nulls of the MuSiC spectrum will be.

5. CONCLUSIONS

In this paper a new algorithm is proposed for estimating the mutual coupling between the elements of a planar array. The method employs one pilot source and uses one extra element. The algorithm has been tested by computer simulation studies and has been found to perform satisfactorily.

6. REFERENCES


Figure 2: Spatial configuration of the array elements and the extra element.

Figure 3: Spectrum of the MuSiC algorithm cost function for 2 sources with azimuths of 35° and 65°.

Figure 4: MuSiC spectrums using the estimated MCM with varying distance of the extra element from the origin of coordinates.