Direction-of-Departure Estimation Using Cooperative Beamforming

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Abstract—In this paper a novel approach is proposed for estimating the direction-of-departure (DOD) in a frequency-selective multipath channel, where both the transmitter (Tx) and receiver (Rx) employ an antenna array. In particular, the proposed approach exploits the cooperation between the Tx and Rx beamformers, such that the Tx beamformer rotates its mainlobe in a synchronized manner that is known to the Rx. This operation allows the DODs of the multiple paths to be estimated at the Rx by making a set of power measurements. The performance of the proposed approach is investigated using computer simulation studies.

I. INTRODUCTION

It is well-known that the employment of antenna arrays [1] can significantly improve system performance. In arrayed systems, direction-of-departure (DOD) is an important parameter for the Tx beamformer which steers the radiating power towards a particular direction in order to improve the link quality and mitigate interference. The maximum gain occurs when the transmission is aligned along with the direction of the channel [2], i.e. DOD. To be able to form a beam to steer the signal, the Tx needs to know the DOD. Motivated by this fundamental view, a simple and effective approach for estimating DOD is proposed in this paper.

A typical scenario is illustrated with two reflected paths in Fig. 1, where the DODs $\theta_{1}^{(Tx)}$ and $\theta_{2}^{(Tx)}$ are to be estimated in the presence of direction-of-arrivals (DOAs) $\theta_{1}^{(Rx)}$ and $\theta_{2}^{(Rx)}$. A number of DOD estimation approaches are presented in [3]-[7]. In [3] the DOD of a path is estimated using a virtual-cross array at the Tx and a switched linear array at the Rx. The Doppler shift, path delay, DODs and DOAs are built into the impulse response of the channel model. By moving the Tx antennas at a constant speed, the received measurements are processed using the ESPRIT algorithm. The post-processing consecutively removes the Doppler effects, estimates the path delays and DOAs and, finally the DODs. This double-directional channel model [3] is then used in [4] but with a different strategy that a switched multibeam antenna is employed at the Tx. The idea of sequential estimation [3] is also employed in [5] for DOD estimation. However, the above approaches are limited to linear antenna array geometries.

In this paper, a novel approach is proposed based on power estimation at Rx, which is simple and effective. It exploits the concept that when the mainlobes of the Tx and Rx beamformers are respectively steered towards the DOD and DOA of a specific path, then the maximum received power will be observed at the Rx. Hence, “rotating” the Tx beamformer causes a change in the received power, and consequently the DOD can be identified as the particular direction that provides the maximum power estimate at the Rx. This idea is not limited to the array geometry, receiver type or the presence/absence of the line-of-sight.

This paper is organized as follows. In Section II, the received signal is presented using a vector-input vector-output (VIVO) channel model. Then in Section III the proposed cooperative beamforming is presented in four brief steps. In Section IV, the performance of the proposed approach is examined using two different Rx beamformers. In Section V the paper is concluded.
II. SIGNAL MODEL

Consider a Rx with an array of \( N_{\text{Rx}} \) antennas receiving a signal via a \( K \)-path frequency-selective channel from a Tx array of \( N_{\text{Tx}} \) antennas, where the VIVO channel is illustrated by Fig. 2.

\[
\mathbf{z}(t, \theta) = \sum_{k=1}^{K} \sqrt{P_{k}} \beta_{k} \mathbf{S}_{k}^{(\text{Rx})} \mathbf{w}_{k}^{(\text{Tx})}(\theta) m(t - \tau_{k}) + \mathbf{n}(t)
\]

where \( \theta \in \mathcal{D} \) denotes the scanning direction at the Tx and has been added to \( \mathbf{z}(t) \) as a parameter to be estimated. Symbol \( \mathcal{D} \) represents the scanning sector. Complex additive white Gaussian noise (AWGN) vector \( \mathbf{n}(t) \in \mathbb{C}^{N_{\text{Rx}} \times 1} \) is assumed to have zero mean and a covariance matrix \( \mathbf{R}_{n} \). The transmitted baseband signal \( m(t) \) is randomly generated with unit power and \( m(t - \tau_{k}) \) is its delayed version with \( \tau_{k} \) the delay of the \( k \)th path. The complex fading coefficient is denoted by \( \beta_{k} \) for the \( k \)th path and \( P_{k} \) is the transmitted power of the signal. The Tx beamformer, using unity norm weight vector \( \mathbf{w}_{k}^{(\text{Tx})}(\theta) \in \mathbb{C}^{N_{\text{Tx}} \times 1} \), is formed towards a scanning direction \( \theta \). The Tx array manifold vector \( \mathbf{S}_{k}^{(\text{Tx})} \in \mathbb{C}^{N_{\text{Tx}} \times 1} \), for azimuth DOA \( \theta_{k}^{(\text{Tx})} \) of the \( k \)th path, is given by

\[
\mathbf{S}_{k}^{(\text{Tx})} = \mathbf{S}(\theta_{k}^{(\text{Tx})}) = \exp\left(-j2\pi \frac{F_{c}}{c} \mathbf{r}_{k}^{(\text{Tx})} \right)
\]

where \( F_{c} \) is the carrier frequency, \( c \) is the speed of light, \( \mathbf{r}_{k}^{(\text{Tx})} \in \mathbb{R}^{N_{\text{Tx}} \times 3} \) denotes the geometry of the Tx array elements, and \( \mathbf{w}_{k}^{(\text{Tx})} = [\cos \theta_{k}^{(\text{Tx})}, \sin \theta_{k}^{(\text{Tx})}, 0]^{T} \) is the Tx wave-number vector. Similarly, \( \mathbf{S}_{k}^{(\text{Rx})} = \mathbf{S}(\theta_{k}^{(\text{Rx})}) \in \mathbb{C}^{N_{\text{Rx}} \times 1} \) is the Rx array manifold for DOA \( \theta_{k}^{(\text{Rx})} \). Without loss of generality, the elevation angle is assumed equal to zero throughout this paper.

III. PROPOSED COOPERATIVE BEAMFORMING

With reference to Fig. 3-6, the approach of the proposed cooperative beamforming can be described in four steps. As shown in the figures, a representative example of a 2-path channel is used. A feedback link from the Rx to Tx and a perfect synchronization between the Tx and Rx are assumed.

**Step-1:** Firstly, the DOAs of all the paths are estimated. The mainlobe of the Tx rotates with a certain fixed angular velocity over a set of scanning directions \( \theta \in \mathcal{D} \) (for instance, \( \mathcal{D} = \{1^\circ, 2^\circ, \ldots, 180^\circ\} \)). For a particular direction \( \theta \), a Tx beamformer \( \mathbf{w}_{k}^{(\text{Tx})}(\theta) \) is constructed using the Tx manifold vector and normalized to have unity norm

\[
\mathbf{w}_{k}^{(\text{Tx})}(\theta) = \frac{\mathbf{S}_{k}^{(\text{Tx})}(\theta)}{\| \mathbf{S}_{k}^{(\text{Tx})}(\theta) \|}, \quad \forall \theta
\]

With the Tx beamformer steering to different directions, the Rx receives a number of signal samples with respect to each direction \( \theta \) (note that synchronization is assumed between the Tx and Rx). The DOAs and path delays of \( K \) paths can then be estimated based on these received signal using, for instance, super-resolution algorithm for DOA estimation (e.g., MUSIC).

Note that delay estimation algorithms are not considered in this paper. The first step is illustrated in Fig. 3.

**Step-2:** The Rx steers one mainlobe to each estimated DOA by constructing beamformers \( \mathbf{w}_{k}^{(\text{Rx})} \in \mathbb{C}^{N_{\text{Rx}} \times 1} \), \( k = 1, \ldots, K \) (see Fig. 4). There are a number of choices available, e.g. Wiener-Hopf (WH), minimum mean-square error (MMSE) or subspace-type beamformers. Two typical Rx beamformers are presented in Section IV.

**Step-3:** This step is shown in Fig. 5. With a Rx beamformer steered to a particular DOA, for every Tx scanning direction
\( \theta \in \mathcal{D} \), \( L \) snapshots of \( \varphi(t, \theta) \) are collected at the Rx. Note that the velocity of rotations at the Tx is known to the Rx. Then the power \( P_k(\theta) \) at the output of the Rx beamformer is estimated for the \( k \)th path as follows

\[
P_k(\theta) = \frac{1}{L} \sum_{l=1}^{L} \left| \mathbf{w}_k^{(\text{Rx})} \varphi_{(t, \theta)} \right|^2, \quad \forall k, \theta
\]

where \( \mathbf{w}_k^{(\text{Rx})} \) is the Rx beamformers constructed in Step-2 and the subscript \( l = 1, \ldots, L \). Hence, DOD \( \theta_k^{(\text{Tx})} \) can be quickly identified as the scanning direction that yields the maximum power estimate Eq. (4), such that

\[
\theta_k^{(\text{Tx})} = \arg \max_{\theta} P_k(\theta), \quad \forall k, \theta
\]

![Fig. 5. The 3rd step of the proposed approach: Rotate Tx beamformer and estimate the received power at each rotation point then estimate DODs as the directions associated with the largest powers.](image)

**Step-4:** The DOD estimates are fed back to the Tx using a feedback link. With Tx and Rx beamformers steered to DODs and DOAs respectively, the identified paths are utilized by a number of beams. Furthermore, the maximum ratio combining (MRC) approach can be used to combine the paths, therefore improving the overall link quality. The fourth step is shown in Fig. 6 while the implementation procedure of the proposed approach is summarized in Table I.

![Fig. 6. The final step of the proposed approach: Feed back the estimated DODs to Tx and steer mainlobes of the Tx towards these DODs.](image)

**IV. SIMULATION STUDIES**

**A. Rx Beamformer**

Two typical Rx beamformers are considered for comparison, respectively based on WH [8] and the subspace [8] method. The well-known WH beamformer is given by

\[
\mathbf{w}_k^{(\text{Rx})} = \mathbb{R}^{-1}_{xx} \mathbf{S}_k^{(\text{Rx})}
\]

where \( \mathbf{S}_k^{(\text{Rx})} \) is formed based on the estimated \( \theta_k^{(\text{Rx})} \) and the covariance matrix of the received signal is given by \( \mathbb{R}_{xx} = \mathbb{E}\{\varphi(t, \theta) \varphi(t, \theta) \} \in \mathbb{C}^{N_{\text{Rx}} \times N_{\text{Rx}}} \).

![TABLE I

**IMPLEMENTATION OF THE PROPOSED COOPERATIVE BEAMFORMING**

<table>
<thead>
<tr>
<th>Step</th>
<th>Task</th>
</tr>
</thead>
<tbody>
<tr>
<td>1a</td>
<td>rotate ( \mathbf{w}_k^{(\text{Tx})}(\theta) ) using Eq. (3), ( \theta \in \mathcal{D} );</td>
</tr>
<tr>
<td>1b</td>
<td>estimate ( \theta_k^{(\text{Rx})} ), ( \forall k );</td>
</tr>
<tr>
<td>2</td>
<td>steer ( \mathbf{w}_k^{(\text{Rx})} ) using Eq. (6) or Eq. (7), ( \forall k );</td>
</tr>
<tr>
<td>3a</td>
<td>rotate ( \mathbf{w}_k^{(\text{Tx})}(\theta) ) using Eq. (3), ( \forall k );</td>
</tr>
<tr>
<td>3b</td>
<td>calculate ( P_k(\theta) ) at Rx with Eq. (4), ( \forall k, \theta );</td>
</tr>
<tr>
<td>3c</td>
<td>identify ( \theta_k^{(\text{Tx})} ) using Eq. (5), ( \forall k );</td>
</tr>
<tr>
<td>4</td>
<td>feed the estimated DODs to Tx and steer the Tx mainlobes (one per DOD)</td>
</tr>
</tbody>
</table>

The subspace type beamformer is based on the projection operator and isolates (receives) one single path while providing cancellation of the other paths as unwanted interference. This provides, asymptotically, complete self-interference cancellation. This beamformer is defined as

\[
\mathbf{w}_k^{(\text{Rx})} = \mathbb{P}_k^{\perp} \mathbb{S}_k^{(\text{Rx})}
\]

where \( \mathbb{S}_k^{(\text{Rx})} \) is the Rx manifold vector of the \( k \)th path and \( \mathbb{P}_k^{\perp} \) is the complement projection operator onto the subspace spanned by the columns of matrix \( \mathbb{S}_k \). That is

\[
\mathbb{P}_k^{\perp} = \mathbb{I}_{N_{\text{Rx}}} - \mathbb{S}_k \mathbb{S}_k^H \mathbb{S}_k^H - \mathbb{S}_k^H
\]

where \( \mathbb{S}_k \in \mathbb{C}^{N_{\text{Rx}} \times (K-1)} \) is the matrix with the Rx manifold vectors of all paths but the \( k \)th path

\[
\mathbb{S}_k = [\mathbb{S}_1^{(\text{Rx})}, \ldots, \mathbb{S}_{k-1}^{(\text{Rx})}, \mathbb{S}_{k+1}^{(\text{Rx})}, \ldots, \mathbb{S}_K^{(\text{Rx})}], \quad k \geq 2
\]

**B. Simulation Results**

Without any loss of generality, both the Tx and Rx are assumed to use uniform linear antenna arrays of half-wavelength spacing with numbers of elements \((N_{\text{Tx}}, N_{\text{Rx}}) = (4,8)\). Assume that the number of paths is \( K = 3 \), where DODs = \([38^\circ, 62^\circ, 115^\circ]\) and DOAs = \([58^\circ, 112^\circ, 145^\circ]\). Channel fading coefficients are complex random variables with an amplitude of \(|\beta_k| = 0.5\). The baseband transmitted signal is randomly generated with zero mean and \( \mathbb{E}\{\varphi(t)\} = 1 \). The transmitted power is denoted by \( P \) and the channel noise is assumed to be 10 dB below \( P \) (SNR = \( P/\sigma^2 = 10 \) dB). The Tx beamformer rotates with a fixed angular velocity that allows the collection at the Rx of \( L = 500 \) snapshots per scanning direction \( \theta = [1^\circ, 2^\circ, \ldots, 180^\circ] \).

Fig. 7 shows the estimated Tx array patterns and DODs for a 3-path channel, where these Tx array patterns are individually estimated. The DODs are quickly identified as the directions associated with the three peaks. Both WH and the subspace type Rx beamformer give very similar pattern shapes. The standard deviation (STD) of the estimation error vs. SNR and the number of snapshots \( L \) are respectively presented in Fig. 8 and Fig. 9. In the high SNR region (to the right of SNR = 8 dB) of Fig. 8, the STD (in degree) of the subspace type beamformer is smaller than that of the WH beamformer,
samples are collected. The subspace method is able to reach almost zero error when SNR = 15 dB, as shown in Fig. 2, the channel capacity (bit/s/Hz), involving the channel matrix \( H \), is considered as the criterion. In Fig. 9 for a fixed input SNR = 15 dB, as \( L \) increases the subspace beamformer outperforms WH beamformer in all region due to its capability in self-interference cancellation. It is also observed that the subspace method is able to reach almost zero error when 2500 samples are collected.

It is important to examine whether knowing DOD improves the performance of arrayed MIMO communications, thus channel capacity [9], [10] is considered as the criterion. At point-I in Fig. 2, the channel capacity (bit/s/Hz), involving the Tx beamformer, is given by

\[
C \left( \mathbf{w}^{(Tx)} (\theta) \right) / B = \log_2 \det \left\{ \frac{P}{\sigma^2_n} \mathbb{H} \left( I_K \otimes \mathbf{w}^{(Tx)} (\theta) \right) \mathbb{R}_{mm} \right. \\
\left. \cdot \left( I_K \otimes \mathbf{w}^{(Tx)} (\theta) \right)^H \mathbb{H}^H + \mathbb{I}_{N(Rx)} \right\}
\]

where \( B \) denotes the channel bandwidth. The composite channel matrix \( \mathbb{H} = [\mathbb{H}_1, \mathbb{H}_2, \ldots, \mathbb{H}_K] \) contains all the paths.
at the output of Rx beamformer, DOD can be quickly identified between the arrayed Tx and Rx. By measuring the power level estimation is proposed based on exploiting the cooperation strategy. With one Rx beamformer per path the MRC combines which provides the capacity at the output of an MRC receiving simulation, the covariance matrix of the delayed signals is given by

\[
H_k = \beta_k \sum_{i=1}^{K} s_i(Tx)^H, \quad k = 1, \ldots, K.
\]

The covariance matrix of the received signal can be written as

\[
R_{mm} = E\{m(t) m(t)^H\}
\]

where \(m(t) = [m(t - \tau_1), m(t - \tau_2), \ldots, m(t - \tau_K)]^T\). In simulation, the covariance matrix \(R_{mm}\) is assumed to be

\[
R_{mm} = \begin{bmatrix}
1 & 0.014e^{-j2.4^\circ} & 0.014e^{j2.4^\circ} & 0.005e^{-j1.6^\circ} & 0.005e^{j1.6^\circ} \\
0.014e^{j2.4^\circ} & 1 & -0.002e^{-j54.7^\circ} & 1 \\
0.014e^{-j2.4^\circ} & -0.002e^{j54.7^\circ} & 1 \\
0.005e^{-j1.6^\circ} & -0.002e^{j54.7^\circ} & 1
\end{bmatrix}
\]

With the parameters \(P, \sigma_n^2, H\) and \(R_{mm}\) fixed, the capacity as a function of \(\theta\) is shown in Fig. 10. It is observed that the three highest peaks are located close to the desired DODs. This result agrees with the statement earlier at the beginning of Section I from a perspective of channel capacity, that knowing DODs will significantly improve system performance. Furthermore, a similar result, as shown in Fig. 11, can be obtained by using the following equation

\[
C\left(\mathbf{w}^{(Tx)}(\theta)\right) = \log_2 \left(\frac{\det \left(R_{mm}\right)}{\det \left(R_{xx}\right) + \sigma_n^2 B} \right)
\]

which provides the capacity at the output of an MRC receiving strategy. With one Rx beamformer per path the MRC combines the outputs of all Rx beamformers in an optimum way.

V. CONCLUSIONS

In this paper, a simple and effective approach for DOD estimation is proposed based on exploiting the cooperation between the arrayed Tx and Rx. By measuring the power level at the output of Rx beamformer, DOD can be quickly identified as the direction with the largest power. The proposed approach is not limited to the array geometry or the line-of-sight.

APPENDIX

The covariance matrix \(R_{xx}\) of the received signal can be written as

\[
R_{xx} = P H W^{(Tx)}(\theta) R_{mm} W^{(Tx)}(\theta)^H + H + R_{nn}
\]

where \(R_{nn} = \sigma_n^2 I_{N_{(Rx)}}\) denotes the covariance matrix of the noise, and \(W^{(Tx)}(\theta) = I_K \otimes \mathbf{w}^{(Tx)}(\theta)\) is the matrix for Tx beamformer. The channel capacity \(C\left(\mathbf{w}^{(Tx)}(\theta)\right)\) is given by

\[
C\left(\mathbf{w}^{(Tx)}(\theta)\right) = \log_2 \left(\frac{\det \left(R_{mm}\right)}{\det \left(R_{xx}\right) + \sigma_n^2 B} \right)
\]

where \(\det \left(b \mathbf{A}\right) = b^n \det \left(\mathbf{A}\right)\), if \(\mathbf{A} \in \mathbb{C}^{N \times N}\), was used.

REFERENCES