A Blind Array Receiver for Multicarrier DS-CDMA in Fading Channels

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Abstract

A modified MMSE receiver for multicarrier DS-CDMA operating in fading, multipath radio channels is presented. This structure is computationally efficient when the channel is rapidly time-varying. Subspace concepts are applied in a proposed blind implementation and simulations show that the new receiver can produce enhanced performance.

Introduction

Multicarrier direct-sequence code division multiple access (MC-DS-CDMA) systems under frequency selective conditions have been analysed in [1] and [2]. It was shown that MC-DS-CDMA can outperform single carrier systems if the multipath intensity profile is exponentially decaying with time, or if the subcarriers are separated by a frequency greater than the inverse chip period. Consequently, MC-DS-CDMA is of interest for future mobile radio systems.

In [3] an MC-DS-CDMA system using an antenna array receiver is considered, operating in a quasi-stationary environment. Subspace decomposition and the technique of alternating projection are applied to formulate a blind, composite channel vector estimator. These channel estimates are then used in a zero-forcing receiver. In this letter the work in [3] is extended for application in rapidly time-varying environments. Similar to [4] a modified MMSE approach is used so that the interference cancelling section of the receiver does not need to track the fading processes. This reduces the computational complexity of the receiver and has the potential to improve its accuracy. A new blind implementation is proposed and simulation results are provided to indicate its performance.

MMSE receiver for fading channels

With reference to Equation 14 of [3] the multicarrier space-time received signal vector for the $n^{th}$ symbol period is described by

$$\mathbf{x}[n] = \mathbf{H}[n] \mathbf{b}[n] + \mathbf{u}[n],$$

(1)
where \( \mathbf{b}[n] \) contains the previous, current and next transmitted symbols for all \( M \) users, \( \mathbb{H}[n] \) contains the associated time-varying composite channel vectors, and \( \mathbf{n}[n] \) is complex AWGN of power \( \sigma^2 \). For this case, the standard MMSE receiver is

\[
W[n] = \mathbb{H}[n] \left( \mathbb{H}[n]^H \mathbb{H}[n] + \sigma^2 I \right)^{-1}.
\]

(2)

Note that the weight matrix is time-varying because of the rapidly fluctuating composite channel vectors. To avoid evaluating Equation 2 every symbol period, \( W[n] \) can be considered to be constant for the channel coherence time. However, when the radio channels are fast fading the standard MMSE receiver becomes expensive to implement.

In [3] it was shown that the composite channel vector for the \( i \)th user can be decomposed as \( \mathbf{h}_i[n] = \mathbf{C}_i \mathbf{s}_i[n] \), where \( \mathbf{C}_i \) is the code matrix for the \( i \)th user and \( \mathbf{s}_i[n] \) contains the channel coefficients at different delays. Considering there to be \( D \) temporally resolvable paths for each user then a related decomposition of the channel vector is \( \mathbf{h}[n] = \mathbf{V} \mathbf{u}[n] \). Here \( \mathbf{u}[n] \) consists of the stacked non-zero elements of \( \mathbf{s}_i[n] \) and \( \mathbf{V} \) contains the corresponding columns of \( \mathbf{C}_i \) at the \( D \) time delays. Hence, \( \mathbf{x}[n] \) can be written as

\[
\mathbf{x}[n] = \mathbf{V} \mathbf{u}[n] + \mathbf{n}[n],
\]

where the substitution \( \mathbf{u}[n] = \mathbf{U}[n] \mathbf{b}[n] \) has been applied and the following definitions are used

\[
\mathbf{V} = \begin{bmatrix} \mathbb{I} \otimes (\mathbb{J}^T)^L & \mathbb{V}_1 \mathbb{V}_2 \cdots \mathbb{V}_M \end{bmatrix},
\]

(4)

\[
\mathbb{V}_i = \begin{bmatrix} \mathbb{V}_1 \mathbb{V}_2 \cdots \mathbb{V}_M \end{bmatrix},
\]

\[
\mathbf{U}[n] = \text{diag}\{\mathbb{U}[n-1], \mathbb{U}[n], \mathbb{U}[n+1]\},
\]

\[
\mathbb{U}[n] = \text{diag}\{\mathbb{U}_1[n], \mathbb{U}_2[n], \ldots, \mathbb{U}_M[n]\}.
\]

The \( \text{diag}\{\cdot\} \) notation means form a block diagonal matrix and \( \otimes \) is the Kronecker product. Furthermore, \( \mathbb{J} \) is a \( 2L \times 2L \) shift matrix, where \( L \) is the number of temporal samples per symbol. \( \mathbb{J} \) is defined to be a \( 2L-1 \) column identity matrix with an additional top row and right hand column containing zero elements.

A modified MMSE receiver can now be formulated as

\[
W = \mathbf{V} \left( \mathbf{V}^H \mathbf{V} + \sigma^2 \mathbb{R}_{\mathbf{u}\mathbf{u}}^{-1} \right)^{-1}.
\]

(5)

This weight matrix is invariant during fading and is valid whilst the time of arrivals (TOAs) of the temporally resolvable paths remain approximately constant relative to the sampling period. If the output of the modified MMSE receiver is \( \mathbf{z}[n] = \mathbf{W}^H \mathbf{x}[n] \), then decision variables for all symbols are generated by diversity combining the signals from different antennas, paths and subcarriers so that \( \mathbf{d}[n] = \mathbf{U}[n]^H \mathbf{z}[n] \).
Blind implementation

Initially an estimate of each users composite channel vector is found by recursive application of the following equation for the $i^{th}$ user (based on the theorem of alternating projection)

$$\hat{h}_i[n] = P_iP_s[n]\hat{h}_i[n-1],$$

where the initial value of $\hat{h}_i[n]$ is any non-zero vector. The projector $P_i = C_i(C_i^H C_i)^{-1} C_i^H$, where $(\cdot)^{-1}$ denotes the pseudo-inverse, restricts the solution to the subspace spanned by the code matrix, and the time-varying projector $P_s[n] = \tilde{E}_s[n]\tilde{E}_s[n]^H$ restricts the solution to the signal subspace. $\tilde{E}_s[n]$ is an orthonormal basis of the signal subspace provided by a subspace tracking algorithm such as RO-FST [5].

To calculate matrix $V$, the TOAs of the $D$ temporally resolvable paths for each user must be estimated. Correlating $\hat{h}_i[n]$ with the temporal vector for the $k^{th}$ subcarrier at all possible delays will produce an interference free spectrum where peaks correspond to estimated TOAs. We define the temporal vector for paths arriving with an integer delay of $\ell$ sample periods as

$$\mathbf{a}_{ik}[\ell] = \left[ a_{ik}[0, \ell], a_{ik}[1, \ell], \ldots, a_{ik}[L-1, \ell], 0^{T_L} \right].$$

The elements of $\mathbf{a}_{ik}[\ell]$ are given by

$$a_{ik}[m, \ell] = \alpha_i \left[ \frac{m}{qN_sc} \right] \exp(j2\pi F_k (m - \ell) T_s),$$

where $\alpha_i$ is the PN-code, $q$ is the oversampling factor, $N_{sc}$ is the number of subcarriers, $F_k$ is the $k^{th}$ subcarrier frequency, $T_s$ is the sampling period and $\lfloor \cdot \rfloor$ means round down to integer.

To provide diversity, the TOA correlation spectrum is averaged over all subcarriers and antennas. Even greater accuracy is achieved by building up a histogram of the peak positions over a block of $B$ symbols during which TOAs are assumed constant. In practice the block period can be significantly greater than the channel coherence time. Knowledge of the TOAs and $C_i \forall i$ means that $V$ can be generated.

The $i^{th}$ component vector of matrix $\mathbf{U}[n]$ is estimated from

$$\tilde{u}_i[n] = V_i^H \tilde{h}_i[n].$$

All $M$ component vectors of $\mathbf{U}[n]$ are estimated and then matrix $\mathbf{U}[n]$ is generated by applying Equation 4. With estimates of $V$ and $\mathbf{U}[n]$ available, Equation 5 can be formulated and the decision variables calculated. A block diagram of the proposed MMSE receiver is shown in Fig. 1.

Simulation results

In the simulations a carrier frequency of 2 GHz was used and the data rate was set to 1 Mbit/s. Differential QPSK modulation and Gold codes of length seven
were used with three times oversampling at the receiver. For the channel model, the paths associated with each user were generated from a Gaussian pdf with 10° standard deviation and mean equal to the nominal user direction. Path delays were set by a uniform pdf with a delay spread of one symbol period and the Doppler frequencies were taken from Clarke’s spectrum [6].

Receiver performance is shown for a system with two receiving antennas spaced by half a wavelength, two subcarriers, and five active users—all moving at 120 km/h. Each user’s channel was composed of ten paths, arriving at two resolvable time delays. The results in Fig. 2 are for three different receivers: 3D RAKE, performs maximal ratio combining in space, time and frequency; MMSE – Equation 2, evaluated every 100 symbols; Proposed MMSE – Equation 5, evaluated every 1000 symbols. The 3D RAKE performs poorly due to uncancelled interference and the standard MMSE receiver has good performance up to the onset of an error floor due to the channel time-variation and residual interference. In contrast, the proposed receiver has a similar performance to the standard MMSE receiver from 0 to 12 dB $E_b/N_0$ but superior performance at higher SNR levels.

**Conclusion**

An alternative approach to the MMSE receiver for array MC-DS-CDMA has been discussed which is applicable for fast fading channels. The proposed blind receiver formulation needs to be evaluated less often than the standard MMSE receiver yet can provide better performance in the moderate to high SNR region.

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**References**


Figure 1: Proposed blind MMSE receiver for fading channels.

Figure 2: Performance when there are five 120 km/h users.