PCM and PSTN
(Public Switched Telephone Network)

Outline:

- PCM: Bandwidth, Bandwidth Expansion Factor, Quantization, output SNR and Threshold Effects, Differential PCM.
- CCITT recommendations for PCM (24-channels and 30-channels)
- Plesiochronous digital hierarchies (PDH)
- Synchronous digital hierarchies (SONET/SDH)
1. INTRODUCTION

- PCM = sampled quantized values of an analogue signal are transmitted via a sequence of codewords.

  i.e. after sampling & quantization, a \textit{Source Encoder} is used to map the quantized levels (i.e. o/p of quantizer) to \textit{codewords of }$\gamma$ \textit{bits}

  \[ \text{i.e. quantized level } \mapsto \text{codeword of } \gamma \text{ bits} \]

  and, then, a digital modulator is used to trasmit the bits, i.e. PCM system

- There are three popular PCM source encoders (or, in other words, Quantization-levels Encoders).
  - \textbf{Binary Coded Decimal (BCD)} source encoder
  - \textbf{Folded BCD} source encoder
  - \textbf{Gray Code (GC)} source encoder
g (input) $\leftrightarrow$ $g_q$ (output)

$g_q$: occurs at a rate $F_s \frac{\text{samples}}{\text{sec}}$

(N.B.: $F_s \geq 2F_g$)

$Q =$ quantizer levels;

$\gamma = \log_2(Q) \frac{\text{bits}}{\text{level}}$

- **Note:**

  $$\frac{\text{codeword rate (o/p of source encoder)}}{\text{sec}} = \frac{\text{quant. levels rate}}{\text{sec}} = \frac{\text{sampling rate}}{\text{sec}} = F_s = 2F_g$$

- **bit rate:** $r_b = \gamma \cdot F_s$ e.g. for $Q = 16$ levels then $r_b = 4 \cdot F_s \frac{\text{bits}}{\text{sec}}$

  $$\downarrow \text{levels} \quad \downarrow \text{sec}$$

  $$(\text{e.g. transmitted sequ.}=101011001101 \ldots)$$

- **versions of PCM**

  - Differential PCM (DPCM): diff. quantizers
  - Delta Modulation: diff. quants with 2 levels $\pm \Delta$ or $-\Delta$

  are encoded using a single binary digit (DM $\in$ DPCM)

- Others
2. PCM: BANDWIDTH & $\beta$

- we transmit several digits for each quantizer's output level $\Rightarrow B_{PCM} > F_g$
  
  where $\left\{ \frac{B_{PCM}}{F_g} \right\}$ denotes the channel bandwidth
  
  represents the message bandwidth

- PCM Bandwidth
  
  **baseband bandwidth:** $B_{PCM} \geq \frac{\text{channel symbol rate}}{2} \text{ Hz}$
  
  **bandpass bandwidth:** $B_{PCM} \geq \frac{\text{channel symbol rate}}{2} \times 2 \text{ Hz}$

- Note that, by default, the Lower bound of the 'baseband' bandwidth is assumed and used in this course

- bandwidth expansion factor $= \beta = \frac{\text{channel bandwidth}}{\text{message bandwidth}}$

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**Example - Binary PCM**

$$B_{PCM} = \frac{\text{channel symbol rate}}{2} = \frac{\text{bit rate}}{2} = \frac{\gamma F_s}{2} = \gamma F_g \text{ Hz} \Rightarrow B_{PCM} = \gamma F_g$$

$$B_{PCM} = \gamma F_g \Rightarrow \frac{B_{PCM}}{F_g} = \gamma \Rightarrow \beta = \gamma$$
3. The Quantization Process (*output point-A2*)

- at point **A2**: 
  a signal discrete in amplitude and discrete in time.

The blocks up to the point A2, combined, can be considered as a discrete information source where a discrete message at its output is a "level" selected from the output levels of the quantizer.

- **analogue samples** $\mapsto$ finite set of levels

where the symbol $\mapsto$ denotes a "map"

In our case this **mapping** is called **quantizing**

i.e.

\[
g(t) \xrightarrow{\text{SAMPLER}} \text{FS} \geq 2F_g \xrightarrow{\text{QUANTIZER}} \{g_q(kT_s)\}
\]
• quantizer parameters:

\[
\begin{align*}
Q & : \text{number of levels} \\
b_i & : \text{input levels of the quantizer, with } i = 0,1,\ldots,Q \\
(m_i = \text{lowest level}) & ; \text{ known as quantizer's end-points} \\
m_i & : \text{outputs levels of the quantizer (sampled values after quantization) with } i = 1,\ldots,Q ; \text{ known as output-levels} \\
rule & : \text{connects the input of the quantizer to } m_i
\end{align*}
\]

RULE:

the sampled values \( g(kT_s) \) of an analogue signal \( g(t) \) are converted to one of \( Q \) allowable output-levels \( m_1, m_2,\ldots, m_Q \) according to the rule:

\[
g(kT_s) \mapsto m_i \quad \text{(or equivalently } \quad g_q(kT_s) = m_i \text{)} \\
\text{iff} \quad b_{i-1} \leq g(kT_s) \leq b_i \quad \text{with } b_0 = -\infty, \quad b_Q = +\infty
\]

• quantization noise at each sample instance:

\[
n_q(kT_s) = g_q(kT_s) - g_s(kT_s)
\]

If the power of the quantization noise is small,

\[
i.e. \quad P_{n_q} = \mathcal{E}\{n_q^2(kT_s)\} = \text{small,}
\]

then the quantized signal (i.e. signal at the output of the quantizer) is a good approximation of the original signal.
• **quality of approximation** may be improved by careful choice of $b_i$'s and $m_i$'s and such as a measure of performance is optimized.

  e.g. measure of performance: Signal to quantization Noise power Ratio 
  (notation: $\text{SNR}_q$) 
  
  $\text{SNR}_q = \frac{\text{signal power}}{\text{quant. noise power}} = \frac{P_g}{P_{nq}}$

• Types of quantization:
  - uniform
  - non-uniform
  - differential = uniform, or non-uniform, plus a differential circuit

• **Transfer Function:**
  - uniform quantizer
  - non-uniform quantizer

  for signals with $\text{CF}=$small
  for signals with $\text{CF}=$large
3.1. UNIFORM QUANTIZERS

• Uniform quantizers are appropriate for uncorrelated samples

\[ g(t) \xrightarrow{\text{SAMPLER}} \{g(kT_s)\} \xrightarrow{\text{QUANTIZER}} \{g_q(kT_s)\} \]

\{g(kT_s)\} \downarrow \text{uncorrelated}

• let us change our notation: \(g_q(kT_s)\) to \(g_q\) and \(g(kT_s)\) to \(g\)

• the range of the continuous random variable \(g\) is divided into \(Q\) intervals of equal length \(\Delta\)

• (value of \(g\)) \(\leftrightarrow\) (midpoint of the quantizing interval in which the value of \(g\) falls)

\[ m_i = \frac{b_{i-1} + b_i}{2} \quad \text{for} \quad i = 1, 2, \ldots, Q \]  \hspace{1cm} (1)
• step size: \[ \Delta = \frac{b_0 - b_0}{Q} \] (2)

• rule: \[ g_q = m_i \quad \text{iff} \quad b_{i-1} < g \leq b_i \quad \text{where} \quad \begin{cases} b_i = b_0 + i \cdot \Delta \\ m_i = \frac{b_{i-1} + b_i}{2} \end{cases} \] (3)

\[ g = m \quad \text{iff} \quad q - b_i \leq b_{i-1} \]

for \( i = 1, 2, \ldots, Q \)

**COMMENTS ON UNIFORM QUANTIZER**

◊ Since, in general, \( Q = \text{large} \) \( \Rightarrow \) \( P_{g_q} \approx P_g \equiv \mathcal{E}\{g^2\} \)

◊ Furthermore, large \( Q \) implies that Fidelity of Quantizer \( = \uparrow \) \( (g_q \approx q) \)

◊ \( Q=8-16 \) are just sufficient for good intelligibility of speech;

(but quantizing noise can be easily heard at the background)

voice telephony: minimum 128 levels; \( \text{(i.e. SNR}_q \approx 42\text{dB)} \)

N.B.: 128 levels \( \Rightarrow \) 7-bits to represent each level

\[ \Rightarrow \text{transmission bandwidth} = \uparrow \]
\[ \text{Quantizer} = \text{UNIFORM} \]
\[ \text{pdf of the input signal} = \text{UNIFORM} \]

then \[ \text{SNR}_q = Q^2 = 2^{2\gamma} \] (4)

\[ \diamond \text{Quantization Noise Power:} \quad P_{n_q} = \frac{\Delta^2}{12} \] (5)

\[ \diamond \text{rms value of Quant. Noise = fixed} = \frac{\Delta}{\sqrt{12}} \neq f\{g\} \] (6)

\[ \therefore \text{if } g(t) = \text{small} \quad \text{for extended period of time} \Rightarrow \text{SNR}_q < \text{the design value} \]

\[ \uparrow \text{this phenomenon is obvious} \]
\[ \text{if the signal waveform has} \]
\[ \text{a large CREST FACTOR} \]

\[ \diamond \text{remember: CREST FACTOR} \equiv \frac{\text{peak}}{\text{rms}} \] (8)
By using **variable spacing** ⇒ CREST FACTOR effects = ↓

\[
\text{small spacing near 0 and large spacing at the extremes}
\]

and this leads to **NON-UNIFORM QUANTIZERS**

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### 3.2. NON-UNIFORM QUANTIZERS

- Non-Uniform quantizers are (like unif. quants) appropriate for **uncorrelated samples**

- **step size** = variable \((\Delta_i)\)

- If pdf\(_{i/p}\) ≠ uniform, then non-uniform quantizers yield higher SNR\(_q\) than uniform quantizers

- rms value of \(n_q\) is not constant but depends on the sampled value \(g(kT_s)\) of \(g(t)\)
• rule: \( g_q = m_i \iff b_{i-1} < g \leq b_i \)

where \( b_0 = -\infty, b_Q = +\infty \), \( \Delta_i = b_i - b_{i-1} \) = variable

• example:

max(SNR) NON-UNIFORM QUANTIZERS

\( b_i, m_i \) are chosen to maximize \( SNR_q \) as follows:

since \( Q \) = large \( \Rightarrow P_{g_q} \approx P_g \equiv \mathcal{E}\{g^2\} \Rightarrow SNR_q = \text{max} \) if \( P_{n_q} = \text{min} \)

where \( P_{n_q} = \sum_{i=1}^{Q} b_{i-1} \int_{b_{i-1}}^{b_i} (g - m_i)^2 \cdot \text{pdf}_g \cdot dg \)

Therefore \( \min_{m_i, b_i} P_{n_q} \) \hspace{1cm} (9)

(9) is equivalent to the following two equations:

\[
\begin{align*}
\frac{dP_{n_q}}{db_j} &= 0 \\
\frac{dP_{n_q}}{dm_j} &= 0
\end{align*}
\] \hspace{1cm} (10)

\[
\begin{align*}
(b_j - m_j)^2 \cdot \text{pdf}_g(b_j) - (b_j - m_{j+1})^2 \cdot \text{pdf}_g(b_j) &= 0 \\
-2 \cdot b_j \int_{b_{j-1}}^{b_j} (g - m_j) \cdot \text{pdf}_g(g) \cdot dg &= 0
\end{align*}
\] \hspace{1cm} (11)

In the second branch of Equation-11 the parameter \( m_j \) can be seen as the statistical mean of the \( j^{th} \) quantizer interval
Note:

the above set of equations (i.e. (11)) cannot be solved in **closed form** for a
general pdf. Therefore for a specific pdf an appropriate method is given
below in a step-form:

**METHOD:**

1. choose a $m_1$
2. calculate $b'_i$, $m_i$
3. check if $m_Q$ is the mean of the interval $[b_{Q-1}, \infty]$
   - if yes → STOP
   - else → choose a new $m_1$ and then goto (2)

**A SPECIAL CASE:**

**max(SNR) Non-Uniform Quantizer of a Gaussian Input Signal**

if the input signal has a Gaussian amplitude pdf, that is, $pdf_g = N(0, \sigma_g)$
then it can be proved that:

$$P_{n_q} = 2.2 \sigma_g^2 Q^{-1.96}$$  \hspace{1cm} (12)

*not easy to derive*

In this case the Signal-to-quantization Noise Ratio becomes:

$$SNR_q = \frac{P_{gq}}{P_{n_q}} = \frac{\sigma_g^2}{2.2 \sigma_g^2 Q^{-1.96}} = 0.45 Q^{1.96}$$  \hspace{1cm} (13)
Compander's type non-uniform quantizers (performance independent of CF)

- **NON-UNIFORM QUANTIZER** ≡

  ≡ **SAMPLE COMPRESSION** + **UNIFORM QUANTIZER** + **SAMPLE EXPANDER**

- **COMPRESSOR + EXPANDER** ≡ **COMPANDER**

\[
g \xrightarrow{f} g_c \quad \text{i.e.} \quad g_c = f\{g\} \quad \text{pdf}_{g_c} = \text{uniform} \quad f^{-1} \xrightarrow{g} g_c
\]

- Popular companders: use \textbf{log} compression

(a) Compression characteristic

(b) Expansion characteristic
• Two compression rules ($A$-law and $\mu$-law) which are used in PSTN and provide a $\text{SNR}_q$ independent of signal statistics are given below:

<table>
<thead>
<tr>
<th>$\mu$-law (USA)</th>
<th>$A$-law (EUROPE)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\mu = 100$</td>
<td>$A = 100$</td>
</tr>
<tr>
<td>$\mu = 5$</td>
<td>$A = 2$</td>
</tr>
<tr>
<td>$\mu = 0$</td>
<td>$A = 1$</td>
</tr>
</tbody>
</table>

• In practice

$$ A \approx 87.6 $$

$$ \mu \approx 100 $$

**Compression-Rules (PCM systems)**

<table>
<thead>
<tr>
<th>$\mu$-law</th>
<th>$A$-law</th>
</tr>
</thead>
<tbody>
<tr>
<td>$g_c = \frac{\ln(1+\mu \cdot \frac{g}{g_{max}})}{\ln(1+\mu)} \cdot g_{max}$</td>
<td>$g_c = \begin{cases} \frac{A \cdot \frac{g}{g_{max}}}{1+\ln(A)} \cdot g_{max} &amp; 0 \leq \frac{g}{g_{max}} &lt; \frac{1}{A} \ \frac{1}{A} \leq \frac{g}{g_{max}} &lt; 1 \end{cases}$</td>
</tr>
</tbody>
</table>

where

$g_c =$ compressor's output signal (i.e. input to uniform quantizer)

$g =$ compressor's input signal

$g_{max} =$ maximum value of the signal $g$
The 6dB LAW

uniform quantizer: \( \text{SNR}_q = 4.77 + 6\gamma - 20\log (\text{CF}) \) dB

\( \text{CF} \) remember \( \frac{\text{peak}}{\text{rms}} \)

\( \mu \)-law: \( \text{SNR}_q = 4.77 + 6\gamma - 20\log (\ln (1 + \mu)) \) dB

\( A \)-law: \( \text{SNR}_q = 4.77 + 6\gamma - 20\log (1 + \ln A) \) dB

- REMEMBER the following figure (illustrates the main characteristics of different types of quantizers)
COMMENTS

• uniform & non-uniform quantizers:

  use them when samples are uncorrelated with each other (i.e. the sequence is quantized independently of the values of the preceding samples)

• practical situation:

  the sequence \{g(kT_s)\} consists of samples which are correlated with each other. In such a case use **differential quantizer**.

• Examples:

  **PSTN**

  \[ F_s = 8\text{kHz}, \quad Q=2^8 (A=87.6 \text{ or } \mu=100), \quad \gamma = 8 \text{ bits/level} \]

  i.e. bit rate: \[ r_b = F_s \times \gamma = 8k \times 8 = 64 \text{ kbits/sec} \]

  **Mobile - GSM**

  \[ F_s = 8\text{kHz}, \quad Q=2^{13} \text{ uniform} \Rightarrow \gamma = 13 \text{ bits/level}, \]

  i.e. bit rate: \[ r_b = F_s \times \gamma = 8k \times 13 = 104 \text{ kbits/sec} \]

  which, with a differential circuit, is reduced to \[ r_b = 13 \text{kbits/sec} \]
3.3. DIFFERENTIAL QUANTIZERS

- Differential quantizers are appropriate for **correlated samples** namely they take into account the sample to sample correlation in the quantizing process;

- e.g. Transmitter (Tx) & Receiver (Rx)

  ![Diagram of Tx and Rx with predictor](image)

  - The weights $w$ are estimated based on the autocorr. function of the input
  - **The Tx & Rx predictors should be identical.** Therefore, the Tx transmits also its weights to the Rx (i.e. weights $w$ are transmitted together with the data)

- In practice, the variable being quantized is not $g(kT_s)$ but the variable $d(kT_s)$

  \[
  d(kT_s) = g(kT_s) - \hat{g}(kT_s)
  \]

  where $d(kT_s)$ is the difference between the actual $g(kT_s)$ and the predicted $\hat{g}(kT_s)$

  i.e.

  ![Diagram of DPCM](image)

  - Because $d(kT_s)$ has small variations, to achieve a certain level of performance fewer bits are required. This implies that DPCM can achieve PCM performance levels with lower bit rates.

  - **6dB Law:**

    \[
    \text{SNR}_q = 4.77 + 6\gamma - a \text{ in dB}
    \]

    where $-10\text{dB} < a < 7.77\text{dB}$
A BETTER DIFFERENTIAL QUANTIZER: mse Diff. Quant.

- the largest error reduction occurs when the differential quantizer operates on the differences between \( g(kT_s) \) and the minimum mean square error (min-mse) estimator \( \hat{g}(kT_s) \) of \( g(kT_s) \)

(N.B.: but more hardware)
\[ \hat{g}(kT_s) = \mathbf{w}^T \mathbf{g} \]

where
\[ \mathbf{g} = [\hat{g}(kT_s), \hat{g}((k-2)T_s), \ldots, \hat{g}((k-L)T_s)]^T \]
\[ \mathbf{w} = [w_1, w_2, \ldots, w_L]^T \]

rule:
\[ \begin{cases} 
\text{choose } \mathbf{w} \text{ to minimize } \mathcal{E} \left\{ \left(g(kT_s) - \hat{g}(kT_s)\right)^2 \right\} & \text{.... for the Transmitter} \\
\text{choose } \mathbf{w} \text{ to minimize } \mathcal{E} \left\{ \left(d_q(kT_s) + \hat{g}(kT_s)\right)^2 \right\} & \text{.... for the Receiver} 
\end{cases} \]

\textbf{DIFFERENTIAL QUANTIZERS: Examples}

The power of \( d(kT_s) \) can be found as follows:
\[
\sigma_d^2 = \mathcal{E}\{d^2\} = \mathcal{E}\{g^2(kT_s)\} + \mathcal{E}\{g^2((k-1)T_s)\} - 2\mathcal{E}\{g(kT_s) \cdot g((k-1)T_s)\} \]
\[
= \sigma_g^2 - 2\mathcal{E}\{g(kT_s) \cdot g((k-1)T_s)\} - 2R_{gg}(T_s)
\]
\[
\Rightarrow \sigma_d^2 = 2\sigma_g^2 - 2R_{gg}(T_s) \Rightarrow \frac{\sigma_d^2}{\sigma_g^2} = 2\left(1 - \frac{R_{gg}(T_s)}{\sigma_g^2}\right) \quad (16)
\]
e.g. disadvantages: unrecoverable degradation is introduced by the quantisation process. (Designers task is to keep this to a subjective acceptable level)

• Remember:

1) $\sigma_g^2 = R_{gg}(0)$

2) $\frac{R_{gg}(\tau)}{\sigma_g^2}$ is known as the normalized autocorrelation function

3) DPCM with the same No of bits/sample → generally gives better results than PCM with the same number of bits.
• Example of mse DPCM

![Diagram of mse DPCM](image)

Assume a 4-level quantizer:

<table>
<thead>
<tr>
<th>Input</th>
<th>Output</th>
</tr>
</thead>
<tbody>
<tr>
<td>$+5 \leq \text{input} \leq +255$</td>
<td>$7$</td>
</tr>
<tr>
<td>$0 \leq \text{input} \leq +4$</td>
<td>$1$</td>
</tr>
<tr>
<td>$-4 \leq \text{input} \leq -1$</td>
<td>$-1$</td>
</tr>
<tr>
<td>$-255 \leq \text{input} \leq -5$</td>
<td>$-7$</td>
</tr>
</tbody>
</table>

**INPUT step from 0V to 26V**

<table>
<thead>
<tr>
<th>$A_n$</th>
<th>$E_n-D_{n-1}$</th>
<th>$B_n-A_n-E_n$</th>
<th>$C_n$</th>
<th>$D_n-C_n+En$</th>
<th>$\text{I/P}$</th>
<th>$\text{O/P}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>26</td>
<td>0</td>
<td>26</td>
<td>+7</td>
<td>7</td>
<td></td>
<td></td>
</tr>
<tr>
<td>26</td>
<td>7</td>
<td>19</td>
<td>+7</td>
<td>14</td>
<td></td>
<td></td>
</tr>
<tr>
<td>26</td>
<td>14</td>
<td>12</td>
<td>+7</td>
<td>21</td>
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<td>21</td>
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<td>+7</td>
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4. NOISE EFFECTS in a binary PCM

- It can be proved that the Signal-to-Noise Ratio at the output of a binary Pulse Code Modulation (PCM) system, which employs a BCD encoder/decoder and operates in the presence of noise, is given by the following expression

\[
\text{SNR}_{\text{out}} = \frac{\mathcal{E}\{g_0(t)^2\}}{\mathcal{E}\{n_0(t)^2\} + \mathcal{E}\{n_{q0}(t)^2\}} = \frac{2^{2\gamma}}{1 + 4p_e \cdot 2^{2\gamma}}
\]

where \( p_e = f(\text{type of digital modulator}) = p_e = T\left\{ \sqrt{(1 - \rho) \cdot \text{EUE}} \right\} \)

e.g. if the digital modulator is a PSK-mod. then \( p_e = T\left\{ \sqrt{2 \cdot \text{EUE}} \right\} \)
4.1. THRESHOLD EFFECTS in a binary PCM

- We have seen that: \( \text{SNR}_{\text{out}} = \frac{2^{2\gamma}}{1+4p_e\cdot2^{2\gamma}} \)

- Let us examine the following two cases: \( \text{SNR}_{\text{in}} = \text{high} \) and \( \text{SNR}_{\text{in}} = \text{low} \)

\[
\begin{align*}
\text{i) } & \text{SNR}_{\text{in}} = \text{HIGH} \\
\text{SNR}_{\text{in}} = \text{high} & \Rightarrow p_e = \text{small} \\
& \Rightarrow 1 + 4p_e\cdot2^{2\gamma} \approx 1 \\
& \Rightarrow \text{SNR}_{\text{out}} \approx 2^{2\gamma} \\
& \Rightarrow \text{SNR}_{\text{out}} \approx 6\gamma \text{ dB}
\end{align*}
\]

\[
\begin{align*}
\text{ii) } & \text{SNR}_{\text{in}} = \text{LOW} \\
\text{SNR}_{\text{in}} = \text{low} & \Rightarrow p_e = \text{large} \\
& \Rightarrow 1 + 4p_e\cdot2^{2\gamma} \approx 4p_e\cdot2^{2\gamma} \\
& \Rightarrow \text{SNR}_{\text{out}} \approx \frac{1}{4p_e}
\end{align*}
\]

**THRESHOLD POINT- definition:**

Threshold point is arbitrarily defined as the \( \text{SNR}_{\text{in}} \) at which the \( \text{SNR}_{\text{out}} \) (i.e. \( \frac{2^{2\gamma}}{1+4p_e\cdot2^{2\gamma}} \)) falls 1dB below the maximum \( \text{SNR}_{\text{out}} \) (i.e. 1dB below the value \( 2^{2\gamma} \)).

- By using the above definition it can be shown (...for you ...) that the threshold point occurs when

\[
p_e = \frac{1}{16\cdot2^{2\gamma}}
\]

where \( \gamma \) is the number of bits per level.
4.2. COMMENTS on THRESHOLD EFFECTS

- The onset of threshold in PCM will result in a sudden $\uparrow$ in the output noise power.
- $P_{\text{signal}} = \uparrow \Rightarrow \text{SNR}_{\text{in}} = \uparrow \Rightarrow \text{SNR}_{\text{out}}$ reaches $6\gamma$ dB and becomes independent of $P_{\text{signal}}$.
- Above threshold: increasing signal power $\Rightarrow$ no further improvement in $\text{SNR}_{\text{out}}$.
- The limiting value of $\text{SNR}_{\text{out}}$ depends only on the number of bits $\gamma$ per quantization levels.

5. DIFFERENTIAL PCM (DPCM)

- DPCM = PCM which employs a differential quantizer

i.e. DPCM reduces the correlation that often exists between successive PCM samples

- The CCITT standards $32\frac{\text{kbits}}{\text{sec}}$ DPCM
  - Speech signal - $F_g = 3.2\text{kHz}$
  - $F_s = 8\frac{\text{ksamples}}{\text{sec}}$
  - $Q = 16$ levels (i.e. $\gamma=4\frac{\text{bits}}{\text{level}}$)

- The CCITT standards $64\frac{\text{kbits}}{\text{sec}}$ DPCM
  - Audio signal - $F_g = 7\text{kHz}$
  - $F_s = 16\frac{\text{ksamples}}{\text{sec}}$
  - $Q = 16$ levels (i.e. $\gamma=4\frac{\text{bits}}{\text{level}}$)
Problems of DPCM:

1. **slope overload noise**: occurs when outer quantization level is too small for large input transitions and has to be used repeatedly.

2. **"Oscillation" or granular noise**: occurs when the smallest $Q$-level is not zero. Then, for constant input, the coder output oscillates with amplitude equal to the smallest $Q$-level.

3. **"Edge Busyness" noise**: occurs when repetitive edge waveform is contaminated by noise which causes it to be coded by different sequences of $Q$-levels.

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**6. INTRODUCTION to TELEPHONE NETWORK**

subscriber-A: 1784-382384

subscriber-B: 20759 46266

Note that, as calls are routed through the PSTN, they will be routed (multiplexed) through a **hierarchy of switching centers**.
1960 British Post Office (BPO) (currently BT) had established a 24-ch PCM system with objective the system to be available in 1968. Some of this work become the basis to the formation of a number of CCITT recommendations.

In Europe, the original 24-ch PCM systems, which were designed mainly for up to 32Km transmission routes, have been replaced by 30-ch PCM systems.
There are **two different** CCITT recommendations for PCM. The main differences between these two recommendations are shown in the following table:

<table>
<thead>
<tr>
<th></th>
<th>1st Recommendation</th>
<th>2nd Recommendation</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Package Size</strong></td>
<td>24-channels</td>
<td>30-channels</td>
</tr>
<tr>
<td><strong>Encoding Law</strong></td>
<td>$\mu$-law $\mu=255$ (but they use $\mu=100$), $\gamma=\frac{7}{6}$ bits/sample; $\gamma=8$ bits/sample</td>
<td>$A$-law $A=87.6$ $\gamma=8$ bits/sample</td>
</tr>
<tr>
<td><strong>Frame-Alignment</strong></td>
<td>FA-signal is distributed amongst several frames</td>
<td>FA-word is placed into a separate slot (TS0)</td>
</tr>
<tr>
<td><strong>Signalling Strategies</strong></td>
<td>Signalling information is conveyed within each speech-time-slot</td>
<td>Signalling information for all 30-channels encoded and conveyed in a separate 8-bit TS (TS-16)</td>
</tr>
</tbody>
</table>

That is, **1st CCITT rec. (24-channels PCM)**

$$T_s = \frac{1}{F_s} = 125\mu\text{sec}$$

That is, **2nd CCITT rec. (30-channels PCM)**

$$T_s = \frac{1}{F_s} = 125\mu\text{sec}$$
- **Note:**

  - A-law = better than μ-law (cheaper to produce and easy equipment maintenance, smaller quantization error in particular within the most significant part of the dynamic range).

  - In 24-ch PCM the signalling information is conveyed within each speech time-slot (technique known as bit stealing). Result: a slight reduction in speech-coding performance.
Based on the 24-channels and 30-channels PCM CCITT recommendations (primary multiplex groups) the core telephone network evolved from using Frequency Division Multiplex (FDM) technology to digital transmission and switching.

These two PCM CCITT recommendations have led to two PDH (Plesiochronous digital hierarchies) CCITT recommendations for assembling the TDM telephony data streams from different calls.

Plesiochronous means: "almost synchronous because bits are stuffed into the frames as padding and the calls location varies slightly - jitters - from frame to frame"
### PDH Hierarchy

<table>
<thead>
<tr>
<th>Hierarchical Level</th>
<th>American DS-(x)</th>
<th>European CEPT-(x)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>DS-0 64 kbits/s</td>
<td>CEPT-0 64 kbits/s</td>
</tr>
<tr>
<td>1</td>
<td>DS-1 1,544 kbits/s</td>
<td>CEPT-1 2,048 kbits/s</td>
</tr>
<tr>
<td>2</td>
<td>DS-2 6,312 kbits/s</td>
<td>CEPT-2 8,448 kbits/s</td>
</tr>
<tr>
<td>3</td>
<td>DS-3 44,736 kbits/s</td>
<td>CEPT-3 34,368 kbits/s</td>
</tr>
<tr>
<td>4</td>
<td>DS-4 274,176 kbits/s</td>
<td>CEPT-4 139,264 kbits/s</td>
</tr>
<tr>
<td>5</td>
<td></td>
<td>CEPT-5 565,148 kbits/s</td>
</tr>
</tbody>
</table>

- **The 24-channel PDH TDM CCITT recommendation (DS-\(x\))**
  - [Diagram showing 24-channel PDH TDM CCITT recommendation]

- **The 30-channel PDH TDM CCITT recommendations (CEPT-\(x\))**
  - [Diagram showing 30-channel PDH TDM CCITT recommendations]
Main disadvantage of PDH Networks

- **PDH multiplexing** was designed for point-to-point communications and channels cannot be added to, or extracted from, a higher multiplexing level demultiplexing down and then multiplexing up again, through the entire PDH.

- For instance, to isolate a particular call from DS4, say, it must be demultiplexed to DS1.

- i.e. this is a very complex procedure and needs very expensive equipment at every exchange to demultiplex and multiplex high speed lines.

- American & European Telephone Systems **are incompatible** (therefore very expensive equipment required to translate one format to the other for transatlantic traffic)

- **Solution: SONET/SDH Signal Hierarchy**

SDH (Synchronous Digital Hierarchy)

- The traditional **PDH standards** are based on the DS (USA) and CEPT (Europe) PCM systems (24-channels and 30-channels PCM CCITT recommendation).

- PDH hierarchy is **almost** synchronous (extra bits are inserted into the digital signal stream to bring them to a common rate).

- In 1988 **SDH (Synchronous Digital Hierarchy)** was adopted by ITU and ETSI (European Telecommunications Standards Institute) based on SONET (synchronous optical Networks).

- SDH signals have a common external timing i.e. **SDH is synchronous**.
- The **SDH standards** used in Europe are
  
  **STM-1** which provides 155 Mbits/sec  
  **STM-2** which provides 310 Mbits/sec  
  **STM-3** which provides 465 Mbits/sec  
  **STM-4** which provides 620 Mbits/sec  
  etc (increments of 155 Mbits/sec )

- The most important main standards are **STM-1, STM-4 and STM-16**. These are **commercially available**

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**SONET/SDH Hierarchy**

<table>
<thead>
<tr>
<th>Hierarchical Level</th>
<th>American SONET STS-(x)</th>
<th>European SDH STM-(x)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>STS-3 = 3 \times DS-3</td>
<td>STM-1 = 1 \times CEPT-4</td>
</tr>
<tr>
<td>1</td>
<td>STS-12 = 12 \times DS-3</td>
<td>STM-4 = 4 \times CEPT-4</td>
</tr>
<tr>
<td>2</td>
<td>STS-48 = 48 \times DS-3</td>
<td>STM-16 = 16 \times CEPT-4</td>
</tr>
</tbody>
</table>

**Key Advantages**

- it is **simple to add and drop** channels to meet customer requirements  
- **more bandwidth** is available for network management  
- equipment is smaller and **cheaper**  
- network **flexibility**  
- integrate and manage **various types of traffic** on a single fiber.
PDH Nets
SDH Nets
Mobile Nets
ATM Nets
IP Nets
Intelligent Networks
e tc.
Network Gateways

PO I S
xDSL
2G
3G
BI SDN
bluetooth
ethernet
GUI etc.

Gateway Interface

CORE Network No.1

CORE Network No.2

CORE Network No.3

Access Networks

Access Networks

Access Networks

Access Network No.1

Access Network No.2

Access Network No.3

etc.