Principles of Code Division Multiple Access (CDMA)

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Autumn 2009

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1 Introduction

- General Block Diagram of a Digital Comm. System (DCS)

![Diagram of a digital communication system]

Advanced Communication Theory (E4.01, ISE4.1, MSc C2)

Principles of CDMA

- Most of the current cellular systems, such as GSM, use frequency division multiplex - time division multiplex (FDM-TDM) techniques to improve the system capacity.

![Spectrum diagram]

Advanced Communication Theory (E4.01, ISE4.1, MSc C2)
HSCDS: High Speed Circuit Switched Data
GPRS: General Packet Radio Systems (2+)
EDGE: Enhanced Data Rate GSM Evolution (2+)
UMTS: Universal Mobile Telecommunication Systems (3G)
WCDMA (Wideband CDMA) is a 3G mobile comm system. It is a wireless system where the telecommunications, computing and media industry converge and is based on a Layered Architecture design. (Note: CDMA Systems $\in$ the class of SSS).

Advanced Communication Theory (E4.01, ISE4.1, MSc C2)

1.1 Definition of a SSS

When a DCS becomes a Spread Spectrum Systems (SSS)

- A general Communication System is CONCERNED with the efficient utilization of energy and bandwidth. That is, it is concerned with keeping low the

  $\quad \text{EUE and BUE}$

- A SSS SACRIFICES the efficient utilization of bandwidth. That is, it deliberately increases the

- **Lemma** – 1: $\quad \text{CS} \triangleq \text{SSS} \iff$

  \[
  \begin{align*}
  &\circ B_{ss} \gg \text{message bandwidth (i.e. BUE=large)} \\
  &\circ B_{ss} \neq f(\text{message}) \quad \text{(spread is achieved by means of a code which is} \neq f(\text{message)}) \\
  &\circ B_{ss}=\text{transmitted SS signal bandwidth}
  \end{align*}
  \]

- **our AIM:** ways of accomplishing LEMMA-1.

Advanced Communication Theory (E4.01, ISE4.1, MSc C2)
- NB:
  - PCM, FM, etc spread the signal bandwidth but do not satisfy the conditions to be called SSS

\[ B_{\text{transmitted-signal}} \gg B_{\text{message}} \]

\[ \Rightarrow \text{SSS distributes the transmitted energy over a wide bandwidth} \]

\[ \Rightarrow \text{SNIR at the receiver input is LOW.} \]

Nevertheless, the receiver is capable of operating successfully because the transmitted signal has distinct characteristics relative to the noise

---

Advanced Communication Theory (E4.01, ISE4.1, MSc C2)
• The PN signal \( b(t) \) is a function of a PN sequence of \( \pm 1 \)'s \( \{ \alpha[n] \} \)

- The sequences \( \{ \alpha[n] \} \) must agreed upon in advance by Tx and Rx and they have status of password.

- This implies that:
  * knowledge of \( \{ \alpha[n] \} \) \( \Rightarrow \) demodulation=possible
  * without knowledge of \( \{ \alpha[n] \} \) \( \Rightarrow \) demod.=very difficult

- If \( \{ \alpha[n] \} \) (i.e. “password”) is purely random, with no mathematical structure, then
  * without knowledge of \( \{ \alpha[n] \} \) \( \Rightarrow \) demodulation=impossible

- However all practical random sequences have some periodic structure.
  This means: \( \alpha[n] = \alpha[n+N_c] \) where \( N_c = \) period of sequence
  i.e. pseudo-random sequence (PN-sequence)

Advanced Communication Theory (E4.01, ISE4.1, MSc C2)

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1.2 Classification of SSS

![Diagram showing the classification of SSS](image)

Advanced Communication Theory (E4.01, ISE4.1, MSc C2)
1.3 Modelling of $b(t)$ in SSS

- DS-SSS (Examples: DS-BPSK, DS-QPSK):
  \[ b(t) = \sum_n \alpha[n].c(t - nT_c) \]  
  where \( \{\alpha[n]\} \) is a sequence of \( \pm 1 \)'s; \( c(t) \) is an energy signal of duration \( T_c \)

- FH-SSS (Examples: FH-FSK)
  \[ b(t) = \sum_n \exp \{ j(2\pi k[n]F_1 t + \phi[n]) \}.c(t - nT_c) \]  
  where \( \{k[n]\} \) is a sequence of integers such that
  \[ \{\alpha[n]\} \mapsto \{k[n]\} \]  
  and \( \{\alpha[n]\} \) is a sequence of \( \pm 1 \)'s; \( c(t) \) is an energy signal of duration \( T_c \)

1.4 Applications of Spread Spectrum Techniques

1. Interference Rejection: to achieve interference rejection due to:
   - Jamming (hostile interference). N.B.: protection against cochannel interference is usually called anti-jamming (AJ)
   - Other users (Multiple Access): Spectrum shared by “coordinated “ users.
   - Multipath: Self-Jamming by delayed signal

2. Energy Density Reduction (or Low Probability of Intercept LPI). LPI’ main objectives:
   - to meet international allocations regulations
   - to reduce (minimize) the detectability of a transmitted signal by someone who uses spectral analysis
   - privacy in the presence of other listeners

3. Range or Time Delay Estimation

NB: interference rejection = most important application
1.5 Definition of a Jammer

- Jamming source, or, simply Jammer is defined as follows:

\[ \text{Jammer} \triangleq \text{intentional (hostile) interference} \]

* the jammer has full knowledge of SSS design except the jammer does not have the key to the PN-sequence generator,
* i.e. the jammer may have full knowledge of the SSSystem but it does know the PN sequence used.

Advanced Communication Theory (E4.01, ISE4.1, MSc C2)

1.6 Processing Gain (PG)

- PG: is a measure of the interference rejection capabilities
- definition:

\[
PG \triangleq \frac{B_{ss}}{B} = \frac{1/T_c}{1/T_{cs}} = \frac{T_{cs}}{T_c}
\]  \hspace{1cm} (4)

where \( B = \) bandwidth of the conventional system
- PG is also known as "spreading factor" (SF)
- PG = very important in DS-SSS
- PG \( \neq \) very important in FH-SSS

Advanced Communication Theory (E4.01, ISE4.1, MSc C2)
1.7 Equivalent EUE

- Remember:
  * Jamming source, or, simply Jammer = intentional interference
  * Interfering source = unintentional interference

\[ B_J = qB_{ss}; \quad 0 < q \leq 1 \quad (5) \]

then

\[ \text{EUE}_J = \frac{E_b}{N_J} = \frac{P_sB_J}{P_JR_b} = \frac{P_sqB_{ss}}{P_JB} = PG \times SJR_{in} \times q \quad (6) \]

i.e. \[ \text{EUE}_{equ} = \frac{E_b}{N_0 + N_J} \quad (7) \]

\[ = PG \times SJR_{in} \times q \times \left( \frac{N_0}{N_J} + 1 \right)^{-1} \quad (8) \]

where

\[ SJR_{in} \triangleq \frac{P_s}{P_J} \quad (9) \]
• SS Transmission in the presence of a Jammer

\[
\text{desired spreader:} \quad m(t) \rightarrow b(t) \mod 2 \quad \{a[n]\} \rightarrow \{\alpha[k]\}
\]

\[
\text{jammer’s spreader:} \quad m_j(t) \rightarrow b_j(t) \mod 2 \quad \{a_j[n]\} \rightarrow \{\alpha_j[k]\}
\]

\[
\text{PSD}(f)
\]

\[
\text{B}_{ss} \quad \text{F}_c
\]

\[
\text{PSD}(f)
\]

\[
\text{B}_{ss} \quad \text{F}_c
\]

Advanced Communication Theory (E4.01, ISE4.1, MSc C2)

• SS Reception in the presence of a Jammer

\[
\text{desired de-spreadder:} \quad b(t) \rightarrow \{\alpha[k]\}
\]

\[
\text{PSD}(f)
\]

\[
\text{B}_{ss} \quad \text{F}_c
\]

\[
\text{PSD}(f)
\]

\[
\text{B}_{ss} \quad \text{F}_c
\]

Advanced Communication Theory (E4.01, ISE4.1, MSc C2)
2 Principles of PN-sequences

• PN-codes (or PN-sequences, or spreading codes) are sequences of +1s and -1s (or 1s and 0s) having special correlation properties which are used to distinguish a number of signals occupying the same bandwidth.

• Five Properties of Good PN-sequences:

<table>
<thead>
<tr>
<th>Property</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>Property-1</td>
<td>easy to generate</td>
</tr>
<tr>
<td>Property-2</td>
<td>randomness</td>
</tr>
<tr>
<td>Property-3</td>
<td>long periods</td>
</tr>
<tr>
<td>Property-4</td>
<td>impulse-like auto-correlation functions</td>
</tr>
<tr>
<td>Property-5</td>
<td>low cross-correlation</td>
</tr>
</tbody>
</table>

2.1 Comments on PN-sequences Main Properties

• Comments on Properties 1, 2 & 3

  – Property-1 is easily achieved with the generation of PN sequences by means of shift registers, while

  – Property-2 & Property-3 are achieved by appropriately selecting the feedback connections of the shift registers.
• Comments on Property-4

– to combat multipath, consecutive bits of the code sequences should be uncorrelated.

i.e. code sequences should have impulse-like autocorrelation functions.

Therefore it is desired that the auto-correlation of a PN-sequence is made as small as possible.

– The success of any spread spectrum system relies on certain requirements for PN-codes. Two of these requirements are:

(a) the autocorrelation peak must be sharp and large (maximal) upon synchronisation (i.e. for time shift equal to zero)

(b) the autocorrelation must be minimal (very close to zero) for any time shift different than zero.

Advanced Communication Theory (E4.01, ISE4.1, MSc C2)
The figure below shows a shift register of 5 stages together with a modulo-2 adder. By connecting the stages according to the coefficients of the polynomial $D^5 + D^2 + 1$ an m-sequence of length 31 is generated (output from Q5). The autocorrelation function of this m-sequence signal is shown in the previous page.

![Shift register and modulo-2 adder](image)

Advanced Communication Theory (E4.01, ISE4.1, MSc C2)

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**Comments on Property-5**

- If there are a number of PN-sequences
  \[
  \{ \alpha_1[k] \}, \{ \alpha_2[k] \}, \ldots, \{ \alpha_K[k] \} 
  \]
  \[ (10) \]

then if these code sequences are not totally uncorrelated, there is always an interference component at the output of the receiver which is proportional to the cross-correlation between different code sequences.

- Therefore it is desired that this cross-correlation is made as small as possible.
2.2 An Important "Trade-off"

- There is a trade-off between Properties-4 and 5.

- In a CDMA communication environment there are a number of PN-sequences

  \[ \{ \alpha_1[k] \}, \{ \alpha_2[k] \}, \ldots, \{ \alpha_K[k] \} \]

  of period \( N_c \) which are used to distinguish a number of signals occupying the same bandwidth.

- Therefore, based on these sequences, we should be able to
  - \textbf{combat multipath}
    (which implies that the \textit{auto-correlation} of a PN-sequence \( \{ \alpha_i[k] \} \) should be made \textit{as small as possible})
  - \textbf{remove interference from other users/signals,}
    (which implies that the \textit{cross-correlation} should be made \textit{as small as possible}).

\[
R_{\text{auto}}^2 + R_{\text{cross}}^2 > \text{a constant which is a function of period } N_c \quad (11)
\]

i.e. there is a \textit{trade-off} between the peak autocorrelation and cross-correlation parameters.

Thus, the autocorrelation and cross-correlation functions cannot be both made small simultaneously.

- The design of the code sequences should be therefore very careful.

- N.B.:
  - A code with excellent autocorrelation is the m-sequence.
  - A code that provides a trade-off between auto and cross correlation is the gold-sequence.
3 m-sequences

- m-seq.: widely used in SSS because of their very good autocorrelation properties.
- PN code generator: is periodic
  i.e. the sequence that is produced repeats itself after some period of time

3.1 Definition of m-seq.

- A sequence generated by a linear $m$-stages FB shift register is called a maximal length, a maximal sequence, or simply m-sequence, if its period is

$$N_c = 2^m - 1$$  \hspace{1cm} (12)

(which is the maximum period for the above shift register generator)

- The initial contents of the shift register are called initial conditions.

3.2 Shift Register and Primitive Polynomials

- The period $N_c$ depends on the feedback connections (i.e. coefficients $c_i$) and
  $N_c = \text{max}$, i.e. $N_c = 2^m - 1$, when the characteristic polynomial

$$c(D) = c_mD^m + c_{m-1}D^{m-1} + \ldots + c_1D + c_0 \quad \text{with} \quad c_0 = 1$$  \hspace{1cm} (13)

is a primitive polynomial of degree $m$.

Rule: if $c_i = \begin{cases} 
0 & \Rightarrow \text{no connection} \\
1 & \Rightarrow \text{there is connection} 
\end{cases}$  \hspace{1cm} (14)

- Definition of PRIMITIVE polynomial = very important (see SSS topic Comm Systems E3)
• Some Examples of Primitive Polynomials

<table>
<thead>
<tr>
<th>degree (-m)</th>
<th>polynomial</th>
</tr>
</thead>
<tbody>
<tr>
<td>3</td>
<td>(D^3 + D + 1)</td>
</tr>
<tr>
<td>4</td>
<td>(D^4 + D + 1)</td>
</tr>
<tr>
<td>5</td>
<td>(D^5 + D^2 + 1)</td>
</tr>
<tr>
<td>6</td>
<td>(D^6 + D + 1)</td>
</tr>
<tr>
<td>7</td>
<td>(D^7 + D + 1)</td>
</tr>
</tbody>
</table>

• (see Comm Systems LNs for some tables of irreducible & primitive polynomial over \(GF(2)\))

3.3 Implementation of an \(m\)-sequence

• use a maximal length shift register

  i.e. in order to construct a shift register generator for sequences of any permissible length, it is only necessary to know the coefficients of the primitive polynomial for the corresponding value of \(m\)

\[
f_c = \frac{1}{T_c} = \text{chip-rate} = \text{clock-rate}
\]  

(15)
\[ c(D) = c_m D^m + c_{m-1} D^{m-1} + \ldots + c_1 D + c_0; \text{ with } c_0 = 1 \]  
\[ (16) \]

- Note that the sequence of 0's and 1's is transformed to a sequence of ±1s by using the following function
\[ o/p = 1 - 2 \times i/p \]  
\[ (17) \]

**Example:**
\[ c(D) = D^3 + D + 1 = \text{primitive} \iff \text{coefficient} = (1, 0, 1, 1) \iff \text{power} = m = 3 \]

<table>
<thead>
<tr>
<th>initial condition</th>
<th>1st</th>
<th>2nd</th>
<th>3rd</th>
</tr>
</thead>
<tbody>
<tr>
<td>clock pulse No.1</td>
<td>0</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>clock pulse No.2</td>
<td>0</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>clock pulse No.3</td>
<td>1</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>clock pulse No.4</td>
<td>0</td>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>clock pulse No.5</td>
<td>1</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>clock pulse No.6</td>
<td>1</td>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>clock pulse No.7</td>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
</tbody>
</table>

\[ N_c = 7 = 2^3 - 1 \quad \text{i.e. period} = 7T_c \]
3.4 Autocorrelation of ‘m-sequences’

- An m-seq. \( \{ \alpha[n] \} \) has a two valued auto-correlation function:

\[
R_{\alpha\alpha}[k] = \sum_{n=1}^{N_c} \alpha[n] \alpha[n + k] = \begin{cases} 
N_c & k = 0 \mod N_c \\
-1 & k \neq 0 \mod N_c 
\end{cases}
\] (18)

- This implies that \( R_{bb}(\tau) \) is also a "two-valued"

\[
R_{bb}(\tau):
\]

- Remember that a sequence \( \{ \alpha[n] \} \) of period \( N_c = 2^m - 1 \), generated by a linear FB shift register, is called a maximal length sequence.

Advanced Communication Theory (E4.01, ISE4.1, MSc C2)

4 Gold Sequences

- Although \( m \)-sequences possess excellent randomness (and especially autocorrelation) properties, they are not generally used for CDMA purposes as it is difficult to find a set of \( m \)-sequences with low cross-correlation for all possible pairs of sequences within the set.

- However, by slightly relaxing the conditions on the autocorrelation function, we can obtain a family of code sequences with lower cross-correlation.

- Such an encoding family can be achieved by Gold sequences or Gold codes which are generated by the modulo-2 sum of two \( m \)-sequences of equal period.
- The Gold sequence is actually obtained by the modulo-2 sum of two $m$-sequences with different phase shifts for the first $m$-sequence relative to the second. Since there are $N_c = 2^m - 1$ different relative phase shifts, and since we can also have the two $m$-sequences alone, the actual number of different Gold-sequences that can be generated by this procedure is $2^m + 1$.

- These sequences, however, are not maximal length sequences. Therefore, their auto-correlation function is not the two valued one given by Equ. (18). The auto-correlation still has the periodic peaks, but between the peaks the auto-correlation is no longer flat.

Advanced Communication Theory (E4.01, ISE4.1, MSc C2)
5 Basics of CDMA

- BLOCK DIAGRAM

Example: DS-BPSK CDMA System

- $K$-users/Transmitters

Multiple Access Channel

SISO = Scalar-Input Scalar-Output Channel
• SISO Multipath channel of the $i$-th user

\[ i/p \xrightarrow{\tau_{i1}} \tau_{i2} - \tau_{i1} \xrightarrow{\tau_{i3} - \tau_{i2}} \text{etc.} \]

\[ \tau_{i1}, \beta_{i1}, \beta_{i2}, \beta_{iL_i} \rightarrow \text{sum} \rightarrow o/p \]

• In the absence of multipaths the above diagram has only $\tau_{i1}$ and $\beta_{i1}$ terms.

• For the simplicity we will drop the second subscript and we will use $\tau_i$ and $\beta_i$, and thus the BPSK/DS-CDMA in the absence of multipaths may be represented as follows:

Advanced Communication Theory (E4.01, ISE4.1, MSc C2)
5.1 Basic Properties of CDMA Systems

- CDMA is one of the applications of spread spectrum communications which is used in civilian, commercial and military communication.

- Two systems: DS-CDMA (i.e. averaging system) and FH-CDMA (i.e. avoidance system).

In this course only DS-CDMA will be considered.

- Assign a specific PN-code to each user

- PN-code (having the status of ‘password’) acts like a ‘channel’

- DS-CDMA: two main cases
  - PN-signal period = \( N_c T_c = T_{cs} \) (known as ‘short codes’ CDMA)
  - PN-signal period = \( N_c T_c \gg T_{cs} \) (known as ‘long codes’ CDMA)

Advanced Communication Theory (E4.01, ISE4.1, MSc C2)
5.2 DS-CDMA: Synchronization

- The Rx requires a replica of the PN code, with the correct clock phase, in order to despread the signal.

- Therefore, Rx = "synchronization circuits" + "demod. circuits"

- The process of synchronizing the receiver to the transmitter’s PN code consists of two stages:
  - Acquisition (coarse synchronization).
  - Tracking (fine synchronization).

![Diagram of Acquisition, Tracking, and Demodulation]

- Operation: acquisition; tracking + demodulation; loose tracking; acquisition; tracking+demodulation; ......etc........
5.3 Mobile Cellular Systems: Conventional & CDMA

- A mobile cellular system consists of base stations, cells (a cell is the area serviced by a base station) and mobiles (subscribers). When a call originates, the base station negotiates with the mobile on various aspects (such as the channel used etc.), before establishing communications. After this, as the mobile moves from cell-to-cell, the service is handed (hand-off or handover) from one base station to another.

- Only one base station will service a mobile at any one time.

- Note:
  - base station to mobile is known as FORWARD LINK
  - mobile to base station is known as REVERSE LINK

---

Type of channels:

<table>
<thead>
<tr>
<th>UPLINK</th>
<th>DOWNLINK</th>
</tr>
</thead>
<tbody>
<tr>
<td>Traffic Channel</td>
<td>Traffic Channel</td>
</tr>
<tr>
<td>Access Channel</td>
<td>Pilot Channel</td>
</tr>
<tr>
<td></td>
<td>Synchron. Channel</td>
</tr>
<tr>
<td></td>
<td>Paging Channel</td>
</tr>
</tbody>
</table>

- Frequency Division Duplex (FDD) and Time Division Duplex (TDD)
5.3.1 Channel Reuse and Reuse Distance

- There is interference from other cells sharing the same channels. The reuse distance \( D \), in these systems, is determined by the worst case interference situation.

- Current cellular systems = FDMA/TDMA
  Most of the current cellular systems, such as GSM, use frequency division multiplex - time division multiplex (FDM-TDM) technique to improve the system capacity. In these systems, each user is assigned one time-frequency slot.
  * When the system gets larger, slots \( \neq \) unique for each and every user
    as this will limit the system capacity. Therefore these slots (time/frequency) have to be reused (reused in cells separated by \( D \) (cells), which is the reuse distance of the system).

---

• The system capacity could be increased by increasing the number of channels available in a single cell, i.e. reducing the reuse distance \( D \).

  But this reduction is limited by the co-channel interference, (i.e. the interference from other cells sharing the same channels).

  The reuse distance \( D \), in these systems, is determined by the worst case interference situation.
• In a CDMA system, the available spectrum and time are not split into distinct slots. Instead the whole (available) spectrum is used by each user.

• Since the same frequency channel could be used by all the users/subscribers, the reuse distance $D$ could be reduced to 1, i.e.

$$\text{if CDMA then } D = 1$$  \hfill (19)

5.3.2 Signal Overlay:

• The spread spectrum signal, from a CDMA system, has a very low power spectral density and, therefore, a CDMA system can overlay on top of existing narrow-band mobile cellular systems (of the same frequency band).

• This is because the interference (due to CDMA signals), added to a narrow-band mobile system channel, is very low and, therefore, the presence of CDMA signal will hardly affect the performance of the narrow-band mobile system.

• The CDMA system, however, needs to perform some extra processing to reject the narrowband interference due to the presence of the narrow-band signals.
6 Analysis of a Direct Sequence CDMA System

- Two systems will be analysed
  * DS/BPSK
  * DS/QPSK

- objective:
  to relate the $p_e$ with the total number $K$ of users as well as with the $EUE_{equ}$ at the receiver.

  i.e.

  $$p_e = f\{EUE_{equ}, K\}$$  \hspace{1cm} (20)
6.1 Main Assumptions

- single cell system of $K$ users,
- $\not\exists$ multipaths
- PN code period $= N_c = PG$
- System=perfectly power-controlled
  (all SS signals arrive at the receiver with the same power)
  NB: power control can often be implemented in practice with great accuracy.
- System = totally asynchronous (there is no common timing reference for the transmitters/users)
  * NB: This is actually an advantage of CDMA over other multiple access techniques, because all users can transmit independently and no signalling information is required.

6.2 DS/BPSK CDMA System: Modelling and Analysis

- Note that the carrier of $i$th transmitter is $\sqrt{P_i} \exp(j(2\pi F_c t + \psi_i))$
Principles of CDMA

- \( i \)-th user’s data signal \( m_i(t) \) and PN-signal \( b_i(t) \):

\[
\begin{align*}
  m_i(t) & \equiv \sum_n a_i[n]. c_1(t - n.T_{cs}) \\
  b_i(t) & = \sum_k \alpha_i[nM + k]. c_2(t - (nM + k)T_c)
\end{align*}
\]  

(21)

- The period of each user’s PN-sequence is selected as \( N_c = \frac{T_{cs}}{T_c} \), and therefore there is one code period per data bit (or \( N_c \) chips per bit). Thus, for the BPSK case, the processing gain PG is:

\[
PG = N_c = \frac{T_{cs}}{T_c}
\]  

(22)

- The transmitted signal \( s_i(t) \) of the \( i \)-th user is therefore

\[
s_i(t) = \sqrt{P_i}.m_i(t).b_i(t). \exp(j(2\pi F_c t + \psi_i))
\]  

(23)

where \( F_c \) is assumed common for all carriers.

Advanced Communication Theory (E4.01, ISE4.1, MSc C2)

---

Principles of CDMA

- Since the transmitters are not time-synchronous, there is a different time delay \( \tau_i \) for each signal \( s_i(t) \) before it reaches the receiver, with \( 0 \leq \tau_i < T_{cs} \) for \( i = 1, 2, 3, \ldots K \). The carrier phases \( \psi_i \) are also assumed different so that \( 0 \leq \psi_i < 2\pi \) for \( i = 1, 2, 3, \ldots K \). Thus, ignoring the band-pass filters at the transmitters and the receiver, the received signal \( r(t) \) can be described as follows:

\[
r(t) = \sum_{i=1}^{K} \frac{\beta_i \sqrt{P_i}}{\Delta \sqrt{P}}.m_i(t - \tau_i).b_i(t - \tau_i). \exp(j(2\pi F_c t + \phi_i)) + n(t)
\]  

(24)

where

\[
\phi_i = \psi_i - 2\pi F_c \tau_i
\]  

(25)

with \( n(t) \) denoting the additive white Gaussian channel noise of double sided power spectral density \( N_0/2 \).
• If the transmitted signals are all synchronized, the delays \( \tau_i, \forall i \), are neglected. Synchronizing all transmitted signals requires a common timing reference and compensation for transmission delays in various transmission paths. This complicates the system requirements and has no clear advantage.

• The receiver has a local PN-signal generator as well as a carrier generator that generate exact replicas of those used in the transmitter. The receiver is in perfect synchronism with the transmitter by using acquisition and tracking techniques (i.e. \( \tau_1, \phi_1 \) are known).

• Thus, without loss of generality, let us assume that \( \tau_1 = 0, \phi_1 = 0 \).

• The output of the 1-st correlator (desired) at \( t = T_{cs} \) is thus

\[
G_1 = \frac{1}{T_{cs}} \int_{(n-1)T_{cs}}^{nT_{cs}} r(t) . b_1(t) . \exp \left( -j(2\pi F_c t) \right) dt
\]

(26)

Advanced Communication Theory (E4.01, ISE4.1, MSc C2)

where, without loss of generality, we have assumed that the 1-st (desired) user is fully synchronised (i.e. \( \tau_1, \phi_1 \) are known) and let us assume \( \tau_1 = 0, \phi_1 = 0 \).

\[
G_1 = \frac{1}{T_{cs}} \int_{(n-1)T_{cs}}^{nT_{cs}} \left\{ \sum_{i=1}^{K} \sqrt{P} . m_i(t - \tau_i) . b_i(t - \tau_i) . \exp(j\phi_i) \right\} . b_1(t) dt
\]

\[
= \frac{1}{T_{cs}} \int_{(n-1)T_{cs}}^{nT_{cs}} \sqrt{P} . m_1(t) . b_1(t) . b_1(t) dt \quad \text{(desired term)}
\]

\[
= \frac{1}{T_{cs}} \sum_{i=2}^{K} \int_{(n-1)T_{cs}}^{nT_{cs}} \sqrt{P} . m_i(t - \tau_i) . b_i(t - \tau_i) . \exp(j\phi_i) . b_1(t) dt \quad \text{(MAI term)}
\]

\[
= \frac{1}{T_{cs}} \int_{(n-1)T_{cs}}^{nT_{cs}} n(t) . b_1(t) . \exp \left( -j2\pi F_c t \right) dt \quad \text{(noise term)}
\]

(27)

Advanced Communication Theory (E4.01, ISE4.1, MSc C2)
The SNIR at the output of the desired receiver can be found to be (for proof see Appendix-1, page 93)

\[
\text{SNIR}_{\text{out}} = \frac{P}{\frac{N_0}{2T_{cs}} + \frac{P}{T_{cs}^2} \sum_{i=2}^{K} \int_0^{T_{cs}} \left\{ R_{b_i b_1}(\tau_i) + \tilde{R}_{b_i b_1}(\tau_i) \right\} d\tau_i}
\]  

(28)

where

\[
\begin{align*}
R_{b_i b_1}(\tau_i) &= \int_0^{\tau_i} b_i(t - \tau_i) b_1(t) dt & \text{even cross-corr.} \\
\tilde{R}_{b_i b_1}(\tau_i) &= \int_{\tau_i}^{T_{cs}} b_i(t - \tau_i) b_1(t) dt & \text{odd cross-corr.}
\end{align*}
\]  

(29)

It is obvious from Eq. 28 that the contribution of the \(i\)-th interfering signal on to \(\text{SNIR}_{\text{out}}\), is inversely proportional to the (even/odd) periodic cross-correlation functions between the \(i\)-th and desired-user's code waveforms. Therefore, designing the code waveforms so that these cross-correlation functions are as small as possible is essential for reducing the total other-user interference (known as multiple access interference) and thus enhancing the system performance.

---

Principles of CDMA

1-st user (desired)

\[ a_1[n-1] \quad a_1[n] \quad a_1[n] \]

\( (n-1)^\text{th} \quad n^\text{th} \quad (n+1)^\text{th} \)

\( T_{cs} \quad T_{cs} \quad T_{cs} \)

\[ \]

\[ \]

\[ \]

\[ \]

i-th user (MAI)

\[ a_i[n-1] \quad a_i[n] \quad a_i[n] \]

\( (n-1)^\text{th} \quad n^\text{th} \quad (n+1)^\text{th} \)

\( T_{cs} \quad T_{cs} \quad T_{cs} \)

\[ \]

\[ \]

\[ \]

\[ \]

Advanced Communication Theory (E4.01, ISE4.1, MSc C2)
6.2.1 SNIR\textsubscript{out} as a function of EUE, $N_c$ and $K$

Equation 28 may be rewritten as:

\[
\text{SNIR}_{\text{out}} = \left(\frac{N_0}{2P_{Tcs}} + \frac{1}{T_{cs}} \sum_{i=2}^{K} \int_{0}^{T_{cs}} \left\{ R_{b_i}^2(\tau_i) + \tilde{R}_{b_i}^2(\tau_i) \right\} d\tau_i \right)^{-1}
\]

\[
= \text{for you} ...
\]

\[
\approx \left\{ \frac{K - 1}{3N_c} + \frac{1}{2 \text{ EUE}} \right\}^{-1}
\]

Advanced Communication Theory (E4.01, ISE4.1, MSc C2)

6.2.2 BER as a function of EUE, $N_c$ and $K$

- We have seen that a good approximation to Equation 28 is:

\[
\text{SNIR}_{\text{out}} \approx \left\{ \frac{K - 1}{3N_c} + \frac{1}{2 \text{ EUE}} \right\}^{-1}
\]  \hspace{1cm} (30)

However,

\[
p_e = T \left\{ \sqrt{\text{SNIR}_{\text{out}}} \right\}
\]  \hspace{1cm} (31)

Therefore,

\[
p_e = T \left\{ \frac{1}{\sqrt{\frac{K - 1}{3N_c} + \frac{1}{2 \text{ EUE}}}} \right\}
\]  \hspace{1cm} (32)

Advanced Communication Theory (E4.01, ISE4.1, MSc C2)
Alternatively by using the concept of $EUE_{equ}$ we have

$$N_j = (K - 1) \cdot \frac{P}{B_{ss}} = (K - 1) \cdot \frac{P \cdot T_c}{N_c} = \frac{(K - 1) \cdot E_b}{N_c}$$

which gives us the following $\text{SNIR}_{out}$ (not necessarily equal to Equation-28),

$$\text{SNIR}_{out} = 2 \cdot EUE_{equ} = 2 \cdot \frac{E_b}{(K-1)E_b + N_0} = \frac{1}{\frac{K-1}{2N_c} + \frac{1}{2 \cdot EUE}}$$

which implies that

$$p_e = T \left\{ \sqrt{2 \cdot EUE_{equ}} \right\} = T \left\{ \frac{1}{\sqrt{\frac{K-1}{2N_c} + \frac{1}{2 \cdot EUE}}} \right\}$$

### 6.2.3 BPSK Examples

- $m_j(t)$
- $b_j(t)$
- $w_j(t) \cdot b_j(t)$
- $A_c \cdot \cos(2\pi f_c t)$
- $S_j(t)$

**BPSK Transmitter ($J^{th}$-user)**
Advanced Communication Theory (E4.01, ISE4.1, MSc C2)
6.3 DS/QPSK CDMA System: Modelling and Analysis

6.3.1 Using Two Different PN-codes per QPSK-user

- The data bits of each user are now grouped in symbols of 2-bits. For instance,

<table>
<thead>
<tr>
<th>input</th>
<th>output</th>
<th>QPSK phase</th>
<th>equivalent ‘symbol’</th>
</tr>
</thead>
<tbody>
<tr>
<td>00</td>
<td>( m_1(t) )</td>
<td>0°</td>
<td>( 1 ) ( \exp(j0°) )</td>
</tr>
<tr>
<td>01</td>
<td>( m_2(t) )</td>
<td>90°</td>
<td>( j ) ( \exp(j90°) )</td>
</tr>
<tr>
<td>11</td>
<td>( m_3(t) )</td>
<td>180°</td>
<td>( -1 ) ( \exp(j180°) )</td>
</tr>
<tr>
<td>10</td>
<td>( m_4(t) )</td>
<td>270°</td>
<td>( -j ) ( \exp(j270°) )</td>
</tr>
</tbody>
</table>

and the \( i \)-th user’s binary information may be now represented by two different data signals \( \dot{m}_{1i}(t) \) and \( \dot{m}_{Qi}(t) \) and two different PN-signals \( \dot{b}_{1i}(t) \) and \( \dot{b}_{Qi}(t) \), as follows:
\begin{align*}
\text{\textbullet \ } \text{i-th user's data signals and PN-signals:} \\
\begin{align*}
\begin{array}{l}
\text{inphase} \\
\quad \quad m_{Ii}(t) & \equiv \sum_{n \text{ (even)}} a_i[n]. c_1(t - n.T_{cs}) \\
\quad \quad b_{Ii}(t) & = \sum_{k \text{ (even)}} \alpha_i[nM + k]. c_2(t - (nM + k)T_c)
\end{array} \\
\begin{array}{l}
\text{quadrature} \\
\quad \quad m_{Qi}(t) & = \sum_{n \text{ (odd)}} a_i[n]. c_1(t - n.T_{cs}) \\
\quad \quad b_{Qi}(t) & = \sum_{k \text{ (odd)}} \alpha_i[nM + k]. c_2(t - (nM + k)T_c)
\end{array}
\end{align*}
\end{align*}
\tag{36}

\textbullet \text{ Note that } m_{Ii}(t) \text{ is formed by the even data bits and } m_{Qi}(t) \text{ by the odd ones.}

\begin{align*}
\text{\textbullet \ } \text{At each transmitter, the two data signals are firstly multiplied by two different PN-code}
\text{ waveforms } b_{Ii}(t) \text{ and } b_{Qi}(t), \text{ and then by}
\sqrt{\frac{P_i}{2}} \exp \left(j \left(2\pi F_c t + \psi_i \right) \right) 
\tag{37}
\end{align*}

while the spread spectrum signal } s_i(t) \text{ of the } i\text{-th user (i.e. transmitted signal) is}
\text{formed by the sum of these two components:}
\begin{align*}
s_i(t) & = \sqrt{\frac{P_i}{2}}(m_{Ii}(t) \cdot b_{Ii}(t) + j m_{Qi}(t) \cdot b_{Qi}(t)) \exp \left(j \left(2\pi F_c t + \psi_i \right) \right) 
\tag{38}
\end{align*}

\text{where } F_c \text{ is assumed common for all carriers.}
Principles of CDMA

- It is important to point out that now the amplitude of the carriers has changed from \( \sqrt{P_i} \) to \( \sqrt{\frac{P_i}{2}} \) so that the power of the transmitted signal \( s_i(t) \) still remains equal to \( P_i \). Indeed

\[
\text{Power of } s_i(t) = \mathcal{E}\{s_i(t)^2\} = \mathcal{E}\{s_i(t).s_i^*(t)\} = 4\text{for you..} = \left( \sqrt{\frac{P_i}{2}} \right)^2 \times 2 = P_i
\]

- Thus the received signal \( r(t) \) can be expressed as

\[
r(t) = \sum_{i=1}^{K} \sqrt{\frac{P_i}{2}} (m_i(t) \cdot b_i(t) + j m_i(t) \cdot b_i(t - \tau_i)) \exp(j(2\pi F_c t + \phi_i)) + n(t)
\]

(39)

where

\[
\sqrt{\frac{P_i}{2}} = \beta_i \sqrt{\frac{P_i}{2}} \quad (40)
\]

\[
\phi_i = \psi_i - 2\pi F_c \tau_i \quad (41)
\]

Advanced Communication Theory (E4.01, ISE4.1, MSc C2)

---

Principles of CDMA

- The received signal is applied at the input of a DS/QPSK1 correlation receiver, matched to the signal \( s_1(t) \).

- The outputs of the I and Q correlators (just before the decision devices) at time \( t = n T_{cs} \) are as follows:

\[
G_{I1} = \frac{1}{T_{cs}} \int_0^{T_{cs}} r(t) \cdot b_{I1}(t) \cdot \exp(-j2\pi F_c t) \, dt
\]

(42)

and

\[
G_{Q1} = \frac{1}{T_{cs}} \int_0^{T_{cs}} r(t) \cdot b_{Q1}(t) \cdot \exp(-j2\pi F_c t) \, dt
\]

(43)

(for proof see Appendix 2, page 102)

Advanced Communication Theory (E4.01, ISE4.1, MSc C2)
• Then it can be found that the SNIR at the output of the inphase branch output of the desired correlator is

\[
\text{SNIR}_{\text{out}} = \frac{P/2}{N_0/2T_{cs} + \frac{P}{2T_{cs}^2} \sum_{i=2}^{K} \int_{0}^{T_{cs}} \left\{ R_{b_1,b_1}(\tau_i) + \tilde{R}_{b_1,b_1}(\tau_i) \right\} d\tau_i + \frac{P}{2T_{cs}^2} \sum_{i=2}^{K} \int_{0}^{T_{cs}} \left\{ R_{b_Q,b_1}(\tau_i) + \tilde{R}_{b_Q,b_1}(\tau_i) \right\} d\tau_i}
\]

(44)

• By symmetry, the SNIR\textsubscript{out} of the quadrature branch is also given by a similar equation.

---

6.3.2 QPSK Example with 2 PN-codes

---
6.3.3 Using One PN-code per QPSK-user

- A simplified version of the above system is a system which uses the same PN-signal for both I & Q branches. The block structure of this system is given below:

*K*-users/Transmitters (QPSK) | Multiple Access Channel | 1-st Receiver

\[ s_i(t) = \sqrt{P_i/2} \exp(j(2\pi f_c t + \psi_i)) \]

\[ m_i(t) \quad QPSK \]

\[ b_i(t) \]

\[ s_i(t) \]

\[ m_k(t) \quad QPSK \]

\[ s_k(t) \]

\[ \beta_i \]

\[ \tau_i \]

\[ \tau_k \]

\[ \beta_K \]

\[ r(t) \]

\[ r(t) \cdot b_{ij}(t) \]

\[ G_i \]

\[ (n\lambda)T_a \]

\[ \exp(-j(2\pi f_c t + \phi_i)) \]

\[ a_i[n] \]
In this case the $\text{SNIR}_{\text{out}}$ (see Equation 44) is simplified to

$$\text{SNIR}_{\text{out}} = \frac{1}{2} \frac{N_0}{2T_{cs}} + \frac{P}{2T_{cs}} \sum_{i=2}^{K} \int_{0}^{T_{cs}} \left\{ R_{b,b_i}^2(\tau_i) + \tilde{R}_{b,b_i}^2(\tau_i) \right\} d\tau_i$$

(45)

By comparing Equ 28 with Equ 45, it is clear that the $\text{SNIR}_{\text{out}}$ for QPSK system is half that of BPSK system.

### 6.3.4 SNIR<sub>out</sub> and BER as a function of EUE, $N_c$ and $K$

- From Equation 45 it can be seen that the $\text{SNIR}_{\text{out}}$ of the QPSK system is half that of the BPSK system. Therefore, an approximation to Equation 45 can be made as

$$\text{SNIR}_{\text{out}} \approx \frac{1}{2} \times \left\{ \frac{K - 1}{3N_c} + \frac{1}{2 \text{EUE}} \right\}^{-1}$$

(46)

i.e.

$$\text{SNIR}_{\text{out}} \approx \left\{ \frac{2(K - 1)}{3N_c} + \frac{1}{\text{EUE}} \right\}^{-1}$$

(47)

- The bit error rate of the inphase branch is related to the $\text{SNIR}_{\text{out}}$ as

$$p_e = T \left\{ \sqrt{\text{SNIR}_{\text{out}}} \right\}$$

(48)

$$\implies p_e = T \left\{ \frac{1}{\sqrt{2\frac{K-1}{3N_c} + \frac{1}{\text{EUE}}}} \right\}$$

(49)
• Alternatively by using the $\text{EUE}_{equ}$ we have

$$N_j = (K - 1) \cdot \frac{P}{B_{ss}} = (K - 1) \cdot P \cdot T_c$$

$$= (K - 1) \cdot \frac{E_b}{N_c}$$

which gives us the following SNIR$_{out}$ (not necessarily equal to Equation-??)

$$\text{SNIR}_{out} = 2 \cdot \text{EUE}_{equ} = 2 \cdot \frac{E_b}{N_c} \cdot \frac{N_0}{N_c} = \frac{1}{2 \cdot \text{EUE}}$$

This implies that

$$p_e = T \left\{ \sqrt{2 \cdot \text{EUE}_{equ}} \right\} = T \left\{ \frac{1}{\sqrt{\frac{K-1}{N_c} + \frac{1}{2 \cdot \text{EUE}}}} \right\}$$

6.3.5 Symbol Error Probability $p_{e,cs}$

• Since, the bit errors in the two branches are statistically independent the probability of symbol error is given as follows

$$p_{e,cs} = 1 - \text{(the joint probability of no bit error in both branches)}$$

$$= 1 - (1 - p_e)^2$$

$$\approx 2 \times p_e$$

$$= (\text{if Equation 49 is used then } ..)$$

$$= 2 \times T \left\{ \frac{1}{\sqrt{\frac{2(K-1)}{3N_c} + \frac{1}{2 \cdot \text{EUE}}}} \right\}$$
6.3.6 QPSK Example with One PN-code.

- The data bits of each user are now grouped in symbols of 2-bits.
- For instance, consider a DS-QPSK SSS with $\phi=45^{\circ}$ (Gray code)

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<th>equivalent ‘symbol’</th>
</tr>
</thead>
<tbody>
<tr>
<td>00</td>
<td>$m_1(t)$</td>
<td>45°</td>
<td>$\frac{\sqrt{2}}{2}(1+j)$</td>
</tr>
<tr>
<td>01</td>
<td>$m_2(t)$</td>
<td>135°</td>
<td>$\frac{\sqrt{2}}{2}(-1+j)$</td>
</tr>
<tr>
<td>11</td>
<td>$m_3(t)$</td>
<td>225°</td>
<td>$\frac{\sqrt{2}}{2}(-1-j)$</td>
</tr>
<tr>
<td>10</td>
<td>$m_4(t)$</td>
<td>315°</td>
<td>$\frac{\sqrt{2}}{2}(+1-j)$</td>
</tr>
</tbody>
</table>

At the Transmitter:

- At the Transmitter:

- The data bits of each user are now grouped in symbols of 2-bits.
- For instance, consider a DS-QPSK SSS with $\phi=45^{\circ}$ (Gray code)

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<th>output</th>
<th>QPSK phase</th>
<th>equivalent ‘symbol’</th>
</tr>
</thead>
<tbody>
<tr>
<td>00</td>
<td>$m_1(t)$</td>
<td>45°</td>
<td>$\frac{\sqrt{2}}{2}(1+j)$</td>
</tr>
<tr>
<td>01</td>
<td>$m_2(t)$</td>
<td>135°</td>
<td>$\frac{\sqrt{2}}{2}(-1+j)$</td>
</tr>
<tr>
<td>11</td>
<td>$m_3(t)$</td>
<td>225°</td>
<td>$\frac{\sqrt{2}}{2}(-1-j)$</td>
</tr>
<tr>
<td>10</td>
<td>$m_4(t)$</td>
<td>315°</td>
<td>$\frac{\sqrt{2}}{2}(+1-j)$</td>
</tr>
</tbody>
</table>
7 Some Important CDMA System Components

7.1 Power Control

- In order to achieve the full benefits of using CDMA, the transmitted signal powers $P_{s_i}$ should be controlled in such a way that received signal power, from all the users at a cell, are the same.

This makes power control a key feature of CDMA mobile systems.
7.2 Voice Activity Factor

- Human speech contains a lot of pauses where there is no data to transmit. Thus a speaker is active for about half the time due to listening and pauses in speech.

The fraction of time that a speaker is active is known as the voice activity factor $a$.

- Extensive studies have shown that $0.35 < a < 0.5$. A popular value used is $a = 3/8 = 0.375$.

- The voice activity feature can be taken into account in a communication system by suppressing the transmission when voice is absent. Assuming that we have a scheme where the carrier is turned-off during the speech idle periods then a reduction in interference (by a factor of the voice activity) can be achieved.
Therefore we can model the “on-off” activity of each user a binomial distribution, which implies that the probability that $k$ users are active is given as follows:

$$\Pr (k \text{ users are active } ) = \binom{k}{K} a^k (1 - a)^{K-k}$$ (54)

where $K$ is the number of users per cell.

Note that as $K \rightarrow \infty$, the spread of distribution $\downarrow$.

Set a threshold $K_{th}, \epsilon_{th}$ such that:

$$\Pr (\text{number of active users } > K_{th} ) < \epsilon_{th}$$ (55)

### 7.3 Sectorization

- Sectorization is achieved by using directional antennas instead of omnidirectional antennas.

- Each cell is divided to three sectors using three directional antennas each having 120° beamwidth.

- Using sectorization the performance can be improved even more.
  
  (The expected value of the total interference is reduced by a factor of 3 wrt single omnidirectional antenna case)

- 

$$\text{SNIR}_{\text{out}} = 2 \cdot \text{EUE}_{eq} = 2 \frac{E_b}{N_j + N_0} = (\text{for you}) =$$ (56)

In practice: 3 dB $< \text{SNIR}_{\text{out}} < 15$ dB
• It is used in both TDMA/FDMA and CDMA systems

• A better approach is to use three linear antenna arrays (smart antennas)
8 Appendices

8.1 Appendix-1: Proof of DS-BPSK/CDMA SNIR\textsubscript{out}

- Proof of SNIR\textsubscript{out} at the 1st Receiver of a BPSK/CDMA System

\[
G_1 = \frac{1}{T_{cs}} \int_{(n-1)T_{cs}}^{nT_{cs}} \left\{ \sum_{i=1}^{K} \sqrt{P.m_i(t - \tau_i)}.b_i(t - \tau_i).\exp(j\phi_i) \right\}.b_1(t) \, dt \\
+ \frac{1}{T_{cs}} \int_{(n-1)T_{cs}}^{nT_{cs}} n(t).b_1(t).\exp(-j2\pi F_c t) \, dt
\]

To simplify the notation \( \int_{(n-1)T_{cs}}^{nT_{cs}} \) will be replaced by \( \int_{0}^{T_{cs}} \) repeated in every interval for any \( n \).
Principles of CDMA

- For the \( i \)-th user, \( a_i[n-1] \) denotes the previous information bit
- \( a_i[n] \) denotes the current information bit
- Note: \( a_i[n-1] \) and \( a_i[n] \) are variables of \( \pm 1 \)'s
- desired term:

\[
\text{desired} = \frac{1}{T_{cs}} \int_0^{T_{cs}} \sqrt{P} m_i(t) b_1^2(t) dt = \frac{\sqrt{P}}{T_{cs}} a_i[n] \int_0^{T_{cs}} b_1^2(t) dt = \sqrt{P} a_i[n] \quad (57)
\]

Advanced Communication Theory (E4.01, ISE4.1, MSc C2)

Principles of CDMA

MAI term:

\[
\text{MAI} = \frac{1}{T_{cs}} \sum_{i=2}^{K} \left\{ \int_{\tau_i}^{\tau_i+\frac{\pm 1}{a_i[n-1]}} \sqrt{P} m_i(t - \tau_i) b_i(t) b_1(t) \exp(j\phi_i) dt \right\} + \int_{\tau_i}^{T_{cs}} \sqrt{P} m_i(t - \tau_i) b_i(t - \tau_i) b_1(t) \exp(j\phi_i) dt \}
\]

\[
= \frac{\sqrt{P}}{T_{cs}} \sum_{i=2}^{K} \left\{ a_i[n-1] \int_{0}^{\tau_i} b_i(t - \tau_i) b_1(t) dt \right\} + a_i[n] \int_{\tau_i}^{T_{cs}} b_i(t - \tau_i) b_1(t) dt \exp(j\phi_i) \}
\]

\[
= \frac{\sqrt{P}}{T_{cs}} \sum_{i=2}^{K} \left\{ a_i[n-1] R_{b_i b_1}(\tau_i) + a_i[n] R_{b_i b_1}(\tau_i) \right\} \exp(j\phi_i) \quad (58)
\]

Advanced Communication Theory (E4.01, ISE4.1, MSc C2)
Principles of CDMA

- where \( R_{b_i b_j}(\tau_i) \) and \( \tilde{R}_{b_i b_j}(\tau_i) \) are the partial cross-correlation functions defined by:

\[
R_{b_i b_j}(\tau_i) = \int_0^{T_i} b_i(t - \tau_i) . b_j(t) \, dt
\]

\[
\tilde{R}_{b_i b_j}(\tau_i) = \int_{\tau_i}^{T_i} b_i(t - \tau_i) . b_j(t) \, dt
\]

- noise term

\[
\text{noise term} = \frac{1}{T_{cs}} \int_0^{T_{cs}} n(t) . b_1(t) . \exp(-j2\pi F_{ct}) \, dt
\]

- and, finally,

\[
G_1 = a_1[n] . \sqrt{P} + \frac{\sqrt{P}}{T_{cs}} \sum_{i=2}^{K} [a_i[n - 1] R_{b_i b_1}(\tau_i) + a_i[n] \tilde{R}_{b_i b_1}(\tau_i)] . \exp(j\phi_i)
\]  

\[
+ \frac{1}{T_{cs}} \int_0^{T_{cs}} n(t) . b_1(t) . \exp(-j2\pi F_{ct}) \, dt
\]

Advanced Communication Theory (E4.01, ISE4.1, MSc C2)

Principles of CDMA

- From Eq 61, it is evident that the first term is the desired signal term, the second term the code noise produced by the cross-correlation with the codes of other users, and the last term the additive white Gaussian noise introduced by the channel.

- It can also be seen that \( G_1 \) is a random variable that is dependent on the mutually independent random variables \( \phi_i, \tau_i, a_i[n - 1] \) and \( a_i[n] \) for \( 1 \leq i \leq K, i \neq j \). To simplify, it is reasonable to assume that the random variables \( \phi_i \) and \( \tau_i \) are uniformly distributed on the intervals \( [0, 2\pi] \) and \( [0, T_{cs}] \) respectively, and \( a_i[n - 1], a_i[n] \) take values \( \pm 1 \) with equal probability.

Advanced Communication Theory (E4.01, ISE4.1, MSc C2)
With the above assumptions, the powers of the three terms in (61) can be found as follows:

- desired power:

\[ P_{\text{desired}} = \mathcal{E} \left\{ \left( \sqrt{P} \cdot a_1[n] \right)^2 \right\} = P \]  

(62)

- MAI power:

\[ P_{\text{MAI}} = \mathcal{E} \left\{ \left( \frac{\sqrt{P}}{T_{cs}} \sum_{i=2}^{K} \left[ a_i[n - 1] R_{b_i b_1}(\tau_i) + a_i[n] \bar{R}_{b_i b_1}(\tau_i) \right] \right)^2 \right\} \]

(63)

\[ = \frac{P}{T_{cs}^2} \sum_{i=2}^{K} \mathcal{E} \left\{ \left( [a_i[n - 1] R_{b_i b_1}(\tau_i) + a_i[n] \bar{R}_{b_i b_1}(\tau_i)] \right)^2 \right\} \]

(64)

Since

\[ \mathcal{E} \{ a_i[n - 1] \} = \mathcal{E} \{ a_i[n] \} = 0 \]  

(65)

\[ \mathcal{E} \{ [a_i[n - 1]]^2 \} = \mathcal{E} \{ [a_i[n]]^2 \} = 1 \]  

(66)

\[ \mathcal{E} \{ \exp(j\phi_i) \} = 1 \]  

(67)

Equation 64 becomes

\[ P_{\text{MAI}} = \frac{P}{T_{cs}^2} \sum_{i=2}^{K} \left\{ \mathcal{E} \left[ a_i[n - 1] R_{b_i b_1}(\tau_i) \right]^2 + \mathcal{E} \left[ a_i[n] \bar{R}_{b_i b_1}(\tau_i) \right]^2 \right\} \]

(68)

- noise term:

\[ P_{\text{noise}} = \mathcal{E} \left\{ \left( \frac{1}{T_{cs}} \int_0^{T_{cs}} n(t).b_1(t).\exp(-j2\pi F c t) \ dt \right)^2 \right\} = \ldots = \frac{N_0}{2T_{cs}} \]  

(69)
Therefore, the SNIR\textsubscript{out} at the output of the correlator is found to be

\[
\text{SNIR}\textsubscript{out} = \frac{P}{\frac{N_0}{2T_{cs}} + \frac{P}{T_{cs}^2} \sum_{i=2}^{K} \int_{0}^{T_{cs}} \left( R_{b_i b_i}^2(\tau_i) + \tilde{R}_{b_i b_i}^2(\tau_i) \right) d\tau_i}
\]  

(70)

N.B.: Note that $F_c \gg \frac{1}{T_{cs}}$. This condition is always satisfied in a practical system.

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8.2 Appendix-2: Proof of DS-QPSK/CDMA SNIR\textsubscript{out}

- Proof of SNIR\textsubscript{out} at the 1st Receiver of a QPSK/CDMA System

- The output of the inphase and quadrature integrators (or correlators) is given as

\[
G_{I1} = \frac{1}{T_{cs}} \int_{(n-1)T_{cs}}^{nT_{cs}} r(t) . b_{I_1}(t) . \exp(-j2\pi F_c t) \, dt 
\]

(71)

\[
G_{Q1} = \frac{1}{T_{cs}} \int_{(n-1)T_{cs}}^{nT_{cs}} r(t) . b_{Q_1}(t) . \exp(-j2\pi F_c t) \, dt 
\]

(72)

- It is only necessary to carry out analysis on one of the branches because of the inherent symmetry. Therefore, the first task is to evaluate the Signal-to-Noise-plus-Interference Ratio (SNIR\textsubscript{out}) at the output of the inphase branch:

- To simplify the notation $\int_{(n-1)T_{cs}}^{nT_{cs}}$ will be replaced by $\int_{0}^{T_{cs}}$ repeated in every interval for any $n$. 

Advanced Communication Theory (E4.01, ISE4.1, MSc C2)
\[ G_{I_i} = \frac{1}{T_{cs}} \int_0^{T_{cs}} \left\{ \sum_{i=1}^{K} \left[ \sqrt{\frac{P}{2}} m_{I_i}(t - \tau_i) \cdot b_{I_i}(t - \tau_i) \\
+ j \sqrt{\frac{P}{2}} m_{Q_i}(t - \tau_i) \cdot b_{Q_i}(t - \tau_i) \cdot \exp(j\theta_i) \right] \right\} \cdot b_{I_1}(t) dt \\
+ \frac{1}{T_{cs}} \int_0^{T_{cs}} n(t) \cdot b_{I_1}(t) \cdot \exp(-j2\pi F_{c}t) dt \\
= \frac{1}{T_{cs}} \int_0^{T_{cs}} \sqrt{\frac{P}{2}} b_{I_1}^2(t) \cdot m_{I_1}(t) \cdot \left( \sum_{i=2}^{K} \left[ \sqrt{\frac{P}{2}} m_{I_i}(t - \tau_i) \cdot b_{I_i}(t - \tau_i) \\
+ j m_{Q_i}(t - \tau_i) \cdot b_{Q_i}(t - \tau_i) \cdot \exp(j\theta_i) \right] \right) \cdot b_{I_1}(t) dt \\
+ \frac{1}{T_{cs}} \int_0^{T_{cs}} n(t) \cdot b_{I_1}(t) \cdot \exp(-j2\pi F_{c}t) dt \] 
(desired Term) 
(MAI Term) 
(noise Term) 
(73)

**Note that** \( b_{I_j}^2(t) = 1 \) and \( m_{I_j}(t) = +1 \ or \ -1 \), after summing over a whole period and dividing by \( T_{cs} \)

**If for the** \( i \)-th user
\[
\begin{align*}
\{ a_{I_i}[n-1], a_{Q_i}[n-1] \} & \quad \text{denotes the previous information bit of} \ I \ or \ Q \\
\{ a_{I_i}[n], a_{Q_i}[n] \} & \quad \text{denotes the current information bit of} \ I \ or \ Q \\
i.e. \ a_{I_i}[n-1], a_{Q_i}[n-1], a_{I_i}[n] \text{ and } a_{Q_i}[n] \text{ are variables of} \ \pm 1 's,
\end{align*}
\]
then Equation 39 can be expressed as follows:

– desire term:
\[
\text{desired} = a_{I_1}[n] \cdot \sqrt{\frac{P}{2}}
\] 
(74)
- MAI term:

\[
\text{MAI} = \frac{\sqrt{P}}{\sqrt{2T_{cs}}} \int_0^{T_{cs}} \left\{ \sum_{i=2}^{K} (m_{Ii}(t - \tau_i) \cdot b_{Ii}(t - \tau_i) \\
+ jm_{Qi}(t - \tau_i) \cdot b_{Qi}(t - \tau_i)) \cdot \exp(j\theta_i) \right\} \cdot b_{I1}(t) \, dt
\]

\[
= \frac{\sqrt{P}}{\sqrt{2T_{cs}}} \sum_{i=2}^{K} \left\{ \int_0^{T_{cs}} (m_{Ii}(t - \tau_i) \cdot b_{Ii}(t - \tau_i) \\
+ jm_{Qi}(t - \tau_i) \cdot b_{Qi}(t - \tau_i)) \cdot \exp(j\theta_i) \cdot b_{I1}(t) \, dt \\
+ \int_{\tau_i}^{T_{cs}} (m_{Ii}(t - \tau_i) \cdot b_{Ii}(t - \tau_i) \\
+ jm_{Qi}(t - \tau_i) \cdot b_{Qi}(t - \tau_i)) \cdot \exp(j\theta_i) \cdot b_{I1}(t) \, dt \right\}
\]

\[
= \frac{\sqrt{P}}{\sqrt{2T_{cs}}} \sum_{i=2}^{K} \left\{ a_{Ii}[n-1]R_{b_{Ii}b_{I1}}(\tau_i) + a_{Ii}[n]R_{\tilde{b}_{Ii}b_{I1}}(\tau_i) \\
+ ja_{Qi}[n-1]R_{b_{Qi}b_{I1}}(\tau_i) + a_{Qi}[n]R_{\tilde{b}_{Qi}b_{I1}}(\tau_i) \right\} \cdot \exp(j\theta_i)
\]

(75)

Advanced Communication Theory (E4.01, ISE4.1, MSc C2)

Principles of CDMA

where \( R_{b_{Ii}b_{I1}}(\tau_i) \), \( \tilde{R}_{b_{Ii}b_{I1}}(\tau_i) \), \( R_{b_{Qi}b_{I1}}(\tau_i) \), \( \tilde{R}_{b_{Qi}b_{I1}}(\tau_i) \) are the partial cross-correlation functions (odd and even) defined as follows:

\[
R_{b_{Ii}b_{I1}}(\tau_i) = \int_0^{\tau_i} b_{Ii}(t - \tau_i) \cdot b_{I1}(t) \, dt
\]

(76)

\[
\tilde{R}_{b_{Ii}b_{I1}}(\tau_i) = \int_{\tau_i}^{T_{cs}} b_{Ii}(t - \tau_i) \cdot b_{I1}(t) \, dt
\]

(77)

\[
R_{b_{Qi}b_{I1}}(\tau_i) = \int_0^{\tau_i} b_{Qi}(t - \tau_i) \cdot b_{I1}(t) \, dt
\]

(78)

\[
\tilde{R}_{b_{Qi}b_{I1}}(\tau_i) = \int_{\tau_i}^{T_{cs}} b_{Qi}(t - \tau_i) \cdot b_{I1}(t) \, dt
\]

(79)

Advanced Communication Theory (E4.01, ISE4.1, MSc C2)
Therefore, finally,

\[
G_{II} = a_{II}[n] \cdot \sqrt{\frac{P}{2}} + \frac{\sqrt{P}}{\sqrt{2T_{cs}}} \sum_{i=2}^{K} \left[ a_{i}[n-1] R_{b_{i},b_{II}}(\tau_i) + a_{Q_i}[n] \tilde{R}_{b_{Q_i},b_{II}}(\tau_i) \right] \cdot \exp(j \theta_i) \\
+ j \frac{\sqrt{P}}{\sqrt{2T_{cs}}} \sum_{i=2}^{K} \left[ a_{Q_i}[n-1] R_{b_{Q_i},b_{II}}(\tau_i) + a_{Q_i}[n] \tilde{R}_{b_{Q_i},b_{II}}(\tau_i) \right] \cdot \exp(j \theta_i) \\
+ \frac{1}{T_{cs}} \int_{0}^{T_{cs}} n(t) \cdot b_{II}(t) \cdot \exp(-j 2 \pi F_c t) \, dt
\] (80)

From Equ 80, it is evident that the first term is the desired signal term, the second and third terms are the code noise produced by the cross-correlation with the codes of other users, and the last term the additive white Gaussian noise introduced by the channel.

It can also be seen that \( G_{IJ} \) is a random variable that is dependent on the mutually independent random variables \( \theta_i, \tau_i, a_{II}[n-1], a_{Q_i}[n-1], a_{I_i}[n] \) and \( a_{Q_i}[n] \) for \( 1 \leq i \leq K, i \neq j \). To simplify, it is reasonable to assume that the random variables \( \theta_i \) and \( \tau_i \) are uniformly distributed on the intervals \([0, 2\pi]\) and \([0, T_{cs}]\) respectively, and \( a_{I_i}[n-1], a_{Q_i}[n-1], a_{I_i}[n] \) and \( a_{Q_i}[n] \) take the values \( \pm 1 \) with equal probability.

With the above assumptions, the mean powers of the four terms in (80) can be found as follows:
Principles of CDMA

- desired

\[ P_{\text{desired}} = \mathcal{E} \left\{ \left( \frac{\sqrt{P}}{2} a_{I_i[n]} \right)^2 \right\} = \frac{P}{2} \]  

(81)

- MAI

\[ P_{\text{MAI}} = \mathcal{E} \left\{ \left( \frac{\sqrt{P}}{\sqrt{2} T_{cs}} \sum_{i=2}^{K} \left[ a_{I_i[n-1]} R_{b_{I_i/I_i}}(\tau_i) + a_{I_i[n]} \tilde{R}_{b_{I_i/I_i}}(\tau_i) \right] \right)^2 \right\} 

+ \mathcal{E} \left\{ \left( \frac{\sqrt{P}}{\sqrt{2} T_{cs}} \sum_{i=2}^{K} \left[ a_{Q_i[n-1]} R_{b_{Q_i/I_i}}(\tau_i) + a_{Q_i[n]} \tilde{R}_{b_{Q_i/I_i}}(\tau_i) \right] \right)^2 \right\} 

= \frac{P}{2 T_{cs}^2} \sum_{i=2}^{K} \left\{ \mathcal{E} \left\{ \left( a_{I_i[n-1]} R_{b_{I_i/I_i}}(\tau_i) + a_{I_i[n]} \tilde{R}_{b_{I_i/I_i}}(\tau_i) \right)^2 \right\} \right\} 

+ \frac{P}{2 T_{cs}^2} \sum_{i=2}^{K} \left\{ \mathcal{E} \left\{ \left( a_{Q_i[n-1]} R_{b_{Q_i/I_i}}(\tau_i) + a_{Q_i[n]} \tilde{R}_{b_{Q_i/I_i}}(\tau_i) \right)^2 \right\} \right\} \]  

(82)

Advanced Communication Theory (E4.01, ISE4.1, MSc C2)

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Principles of CDMA

However,

\[ \mathcal{E} \left\{ [a_{I_i[n-1]]^2} = \mathcal{E} \left\{ [a_{I_i[n]}]^2 \right\} = \mathcal{E} \left\{ [a_{Q_i[n-1]]^2} = \mathcal{E} \left\{ [a_{Q_i[n]}]^2 \right\} = 1, \]  

(83)

\[ \mathcal{E} \left\{ a_{I_i[n-1]} \right\} = \mathcal{E} \left\{ a_{I_i[n]} \right\} = \mathcal{E} \left\{ a_{Q_i[n-1]} \right\} = \mathcal{E} \left\{ a_{Q_i[n]} \right\} = 0 \]  

(84)

also current and previous bits are uncorrelated, i.e.,

\[ \mathcal{E} \left\{ a_{I_i[n-1]} a_{I_i[n]} \right\} = 0 \]  

(85)

\[ \mathcal{E} \left\{ a_{Q_i[n-1]} a_{Q_i[n]} \right\} = 0 \]  

(86)

Furthermore,

\[ \mathcal{E} \left\{ \cos^2(\theta_i) \right\} = \frac{1}{2} \text{ and } \mathcal{E} \left\{ \sin^2(\theta_i) \right\} = \frac{1}{2} \]  

(87)
Therefore, Equation 82 becomes

\[
P_{MAI} = \frac{P}{2T_{cs}^2} \sum_{i=1}^{K} \left( \mathcal{E} \left\{ a_{ii}[n-1] R_{b_i,b_11}(\tau_i) \right\}^2 + \mathcal{E} \left\{ a_{ii}[n] \tilde{R}_{b_i,b_11}(\tau_i) \right\}^2 \right) + \frac{P}{2T_{cs}^2} \sum_{i=1}^{K} \left( \mathcal{E} \left\{ a_{Q_i}[n-1] R_{b_{Qi},b_11}(\tau_i) \right\}^2 + \mathcal{E} \left\{ a_{Q_i}[n] \tilde{R}_{b_{Qi},b_11}(\tau_i) \right\}^2 \right)
\]

\[
= \frac{P}{2T_{cs}^2} \sum_{i=1}^{K} \int_0^{T_{cs}} \left( R_{b_i,b_11}(\tau_i) + \tilde{R}_{b_i,b_11}(\tau_i) \right) d\tau_i \]

\[
+ \frac{P}{2T_{cs}^2} \sum_{i=1}^{K} \int_0^{T_{cs}} \left( R_{b_{Qi},b_11}(\tau_i) + \tilde{R}_{b_{Qi},b_11}(\tau_i) \right) d\tau_i \tag{88}
\]

\[P_{\text{noise}} = \mathcal{E} \left\{ \left( \frac{1}{T_{cs}} \int_0^{T_{cs}} n(t),b_{i1}(t), \exp(-j2\pi F_t t) dt \right)^2 \right\}
\]

\[= \frac{\mathcal{N}_0}{2T_{cs}} \tag{89}
\]

- Therefore, the SNIR\text{out} at the output of the inphase correlator is found to be

\[
\text{SNIR}_{\text{out}} = \frac{P/2}{\frac{\mathcal{N}_0}{2T_{cs}} + \frac{P}{2T_{cs}^2} \sum_{i=2}^{K} \int_0^{T_{cs}} \left( R_{b_i,b_11}(\tau_i) + \tilde{R}_{b_i,b_11}(\tau_i) \right) d\tau_i + \frac{P}{2T_{cs}^2} \sum_{i=2}^{K} \int_0^{T_{cs}} \left( R_{b_{Qi},b_11}(\tau_i) + \tilde{R}_{b_{Qi},b_11}(\tau_i) \right) d\tau_i}
\]