

Study Group

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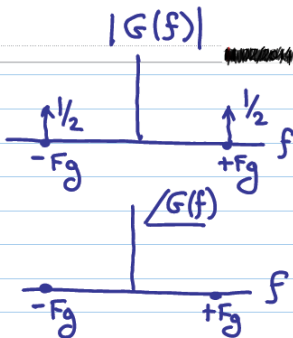
Comms-1

SINEWAVES

PS4[1]

Note Title

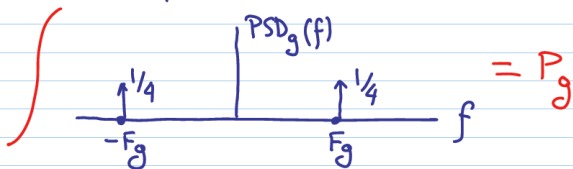
$$g(t) = \cos(2\pi F_g t) \xrightarrow{\text{FS or FT}}$$



$$\text{ie } |G(f)| = \frac{1}{2} \delta(f + F_g) + \frac{1}{2} \delta(f - F_g)$$

$$\angle G(f) = 0$$

$$\begin{aligned} \text{PSD}_g(f) &= G(f) \cdot G(f)^* = |G(f)|^2 = \\ &= \frac{1}{4} \delta(f+F_g) + \frac{1}{4} \delta(f-F_g) \end{aligned}$$



$$\text{ie } P_g = \int \text{PSD}_g(f) df = \frac{1}{4} + \frac{1}{4} = \frac{1}{2}$$

$$\text{indeed } P_g = \underbrace{\int \frac{1}{4} \delta(f+F_g) dg}_{\frac{1}{4}} + \underbrace{\int \frac{1}{4} \delta(f-F_g) dg}_{\frac{1}{4}} = \frac{1}{2}$$

otherwise: $P_g = \overline{g^2(t)} = \overline{\cos^2(2\pi F_g t)} = \frac{1}{2}$

or

, using auto-correlation

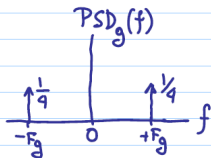
$$\begin{aligned}
 R_{gg}(z) &= \overline{g(t) \cdot g(t-z)} = \\
 &= \overline{\cos(2\pi F_g t) \cos(2\pi F_g (t-z))} \\
 &= \overline{\cos(2\pi F_g t) \cdot [\cos(2\pi F_g t) \cos(2\pi F_g z) + \sin(2\pi F_g t) \sin(2\pi F_g z)]} \\
 &= \overline{\cos^2(2\pi F_g t) \cos(2\pi F_g z) + \overline{\cos(2\pi F_g t) \sin(2\pi F_g t) \cdot \sin(2\pi F_g z)}} \\
 &= \underbrace{\overline{\cos^2(2\pi F_g t)}}_{\frac{1}{2}} \cos(2\pi F_g z) + \underbrace{\overline{\cos(2\pi F_g t) \cdot \sin(2\pi F_g t)}}_0 \cdot \sin(2\pi F_g z) \\
 &= \frac{1}{2} \cos(2\pi F_g z)
 \end{aligned}$$

i.e.

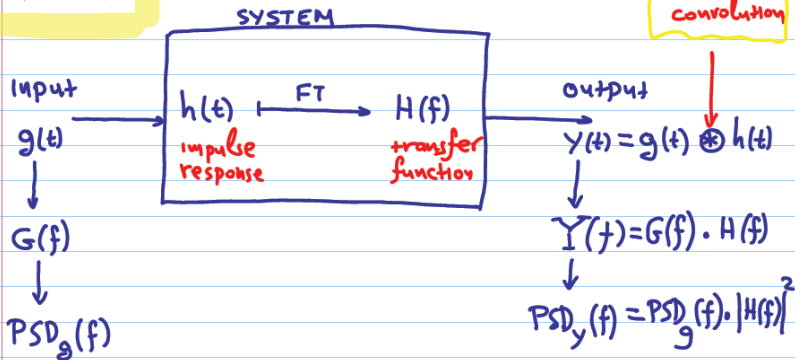
$$R_{gg}(z) = \frac{1}{2} \cos(2\pi F_g z)$$



FT.

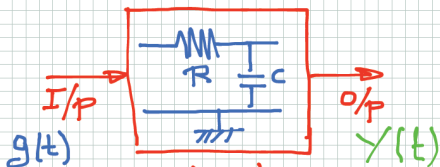


PS 4. [2]



note : $H(f) = \text{complex}$

\swarrow $|H(f)|$
 \searrow $\angle H(f)$

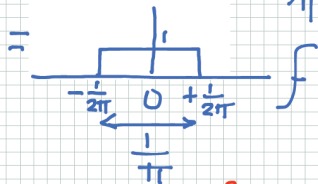


$$H(f) = \frac{1}{1 + j2\pi f}$$

$$\downarrow$$

$$|H(f)|^2 = \frac{1}{1 + (2\pi f)^2}$$

$$PSD_g(f) = \text{rect}(\pi f) = \text{rect}\left(\frac{f}{\frac{1}{2\pi}}\right)$$



$$PSD_y(f) = PSD_g(f) \cdot |H(f)|^2$$

$$\begin{aligned}
 P_y &= \int_{-\infty}^{\infty} \text{PSD}_y(f) df \\
 &= \int_{-\infty}^{\infty} \text{PSD}_g(f) \cdot |H(f)|^2 df \\
 &= \int_{-\infty}^{\infty} \text{rect}(\pi f) \frac{1}{1+(2\pi f)^2} df \\
 &= \int_{-\frac{1}{2\pi}}^{\frac{1}{2\pi}} 1 \cdot \frac{1}{1+(2\pi f)^2} df \\
 &= \frac{1}{2\pi} \tan^{-1}(2\pi f) \Big|_{-\frac{1}{2\pi}}^{\frac{1}{2\pi}} \\
 &= \frac{1}{2\pi} \tan^{-1} 1 - \frac{1}{2\pi} \tan^{-1}(-1) \\
 &= \frac{1}{2\pi} \cdot \frac{\pi}{4} + \frac{1}{2\pi} \cdot \frac{\pi}{4} \\
 &= \frac{1}{4}
 \end{aligned}$$

