Communication Channels

1. If one binary source and two binary channels are connected in cascade as shown below

![Diagram of binary source and channels]

where both channels have the following forward transition probability diagram

![Transition probability diagram]

find the bit-error-rate $p_e$ at the output of the second channel. 10%

2. A digital communication system, operating at 100 bits/sec in the presence of additive white Gaussian noise of power spectral density $PSD_n(f) = \frac{N_0}{2}$, is represented in the energy utilization efficiency (EUE) - bandwidth utilization efficiency (BUE) plane, as follows:

![Energy utilization efficiency and bandwidth utilization efficiency plane]

What is the capacity $C$ of the channel in bits/sec? 20%
3. A digital communication system having an energy utilisation efficiency (EUE) equal to 30 operates in the presence of additive white Gaussian noise of double-sided power spectral density $\text{PSD}_n(f) = 0.5 \times 10^{-6}$ W/Hz. If the channel capacity $C$ is 16 kbits/s and the channel bandwidth $B$ is 4 kHz, estimate

(a) the bit rate $r_b$ 
(b) the noise power at the channel output

4. A discrete channel is modelled as follows:

![Discrete Channel Diagram]

Estimate:

(a) The probability of error at the output of the channel
(b) The amount of information delivered at the output of the channel

5. Consider a binary Communication System that uses the following two equally probable energy signals:

$$0 \rightarrow s_0(t) = 2\Lambda \left\{ \frac{t}{10\mu s} \right\}$$

$$1 \rightarrow s_1(t) = -2\Lambda \left\{ \frac{t}{10\mu s} \right\}$$

The channel is assumed additive white Gaussian noise of double-sided power spectral density $\text{PSD}_n(f) = 10^{-6}$ W/Hz. Find:

(a) the bandwidth $B$ of the channel;
(b) the channel symbol rate $r_{cs}$ (baud rate) & data bit rate;
(c) the Energy Utilisation Efficiency (EUE);
(d) the channel capacity $C$ in bits/sec.

6. Consider a binary Communication System that operates with a bit rate 100 kbits/sec and uses the following two equally probable energy signals:

$$0 \rightarrow s_0(t) = 3 \left\{ \Lambda \left\{ \frac{t}{5\mu s} \right\} + \text{rect} \left\{ \frac{t}{10\mu s} \right\} \right\}$$

$$1 \rightarrow s_1(t) = -3 \left\{ \Lambda \left\{ \frac{t}{5\mu s} \right\} + \text{rect} \left\{ \frac{t}{10\mu s} \right\} \right\}$$

The channel is assumed additive white Gaussian noise of double-sided power spectral density $\text{PSD}_n(f) = 0.5 \times 10^{-6}$ W/Hz. Find:

(a) the bandwidth $B$ of the channel;
(b) the channel symbol rate $r_{cs}$ (baud rate); 5%
(c) the Energy Utilisation Efficiency (EUE); 20%
(d) the channel capacity $C$ in bits/sec. 15%

7. Consider a binary digital communication system in which a binary sequence is transmitted as a signal $s(t)$ with a one being sent as $6A \left\{ t \right\}_{T_{cs}/2}$ and a zero being sent as $-6A \left\{ t \right\}_{T_{cs}/2}$. The source at the input to the system provides a binary sequence of ones and zeros, with the number of ones being twice the number of zeros. The transmitted signal is corrupted by channel noise $n(t)$ of bandwidth $B$ and has an amplitude probability density function described by the following expression:

$$pdf_n(n) = \frac{1}{6} \text{rect} \left\{ \frac{n}{6} \right\}$$

Find a bound on the ratio $C/B$ where $C$ denotes the capacity of the channel in bits/s.

8. Consider a binary digital communication system in which the transmitted signal is corrupted by channel noise of bandwidth $B$ having an amplitude probability density function described by the following expression:

$$pdf_n(n) = \frac{1}{6} \text{rect} \left\{ \frac{n}{6} \right\}$$

If the power of the received signal is 12W then

(a) find the entropy power of the noise; 10%
(b) find an upper and a lower bound on the ratio $C/B$ where $C$ denotes the capacity of the communication channel. 10%

9. A discrete channel is modelled as follows:

Estimate:

(a) The probability of error at the output of the channel 5%
(b) The amount of information delivered at the output of the channel 15%
10. A discrete channel is modelled as follows:

Estimate:

(a) The probability of error at the output of the channel 5%
(b) The amount of information delivered at the output of the channel 15%

11. A signal $g(t)$ bandlimited to 4kHz is sampled at the Nyquist rate and is fed through a 2-level quantizer. A Huffman encoder is used to encode triples of successive output quantization levels as follows:

<table>
<thead>
<tr>
<th>symbols</th>
<th>probs</th>
<th>Huffman</th>
</tr>
</thead>
<tbody>
<tr>
<td>$m_1m_1m_1$</td>
<td>27/64</td>
<td>1</td>
</tr>
<tr>
<td>$m_1m_1m_2$</td>
<td>9/64</td>
<td>001</td>
</tr>
<tr>
<td>$m_1m_2m_1$</td>
<td>9/64</td>
<td>010</td>
</tr>
<tr>
<td>$m_2m_1m_1$</td>
<td>9/64</td>
<td>011</td>
</tr>
<tr>
<td>$m_1m_2m_2$</td>
<td>3/64</td>
<td>00000</td>
</tr>
<tr>
<td>$m_2m_1m_2$</td>
<td>3/64</td>
<td>00001</td>
</tr>
<tr>
<td>$m_2m_2m_1$</td>
<td>3/64</td>
<td>00010</td>
</tr>
<tr>
<td>$m_2m_2m_2$</td>
<td>1/64</td>
<td>00011</td>
</tr>
</tbody>
</table>

while the binary sequence at the output of the Huffman encoder is fed to a Binary on-off Keyed Communication System which employs the following two energy signals of duration $T_{cs}$

$$s_0(t) = 0$$
$$s_1(t) = \sqrt{\frac{3}{8}} \left( \frac{t}{0.5 T_{cs}} \right)$$

The transmitted signals are corrupted by additive white Gaussian channel noise having a double-sided power spectral density of $10^{-3}$ W/Hz. The figure below shows a modelling of the whole system where the output of the Huffman encoder is modelled as the output of a binary discrete information source $(X, p)$ with

$$X = \{x_1 = 1, x_2 = 0\},$$
$$p = [Pr(x_1), Pr(x_2)]^T$$

while the binary on-off Keyed system is modelled as a discrete channel as shown below.
(a) Find the entropy of the information source \((X, P)\), the information rate and the bit data rate (symbol rate) at the channel input. 15%

(b) Estimate the bit-error probability of the system. 10%

(c) Estimate the energy utilization efficiency (EUE) and bandwidth utilization efficiency (BUE) using the bit data rate as well as the information rate. 15%

(d) Represent the communication system, as a point on the (EUE,BUE) parameter plane. In this plane show also the locus of the system properly labelled. 10%

(e) Is the system a ‘realizable’ communication system? 5%

(f) What is the signal-to-noise ratio, \(\text{SNR}_{in}\), at the receiver’s input? 5%

12. A signal \(g(t)\) having the pdf shown in Figure 1 is bandlimited to 4 kHz. The signal is sampled at the Nyquist rate and fed through a 2-level quantizer. The transfer function of the quantizer is shown in Figure 2.

A Huffman encoder is used to encode triples of successive output quantization levels while the binary sequence at the output of the Huffman encoder is fed to a Binary on-off Keyed Communication System which employs the following two energy signals

\[ s_1(t) = 0; s_2(t) = 0.5 \cos \left( \frac{2\pi}{T_{cs}} \frac{5}{T_{cs}} t \right); \text{with } 0 < t < T_{cs} \]

The whole system is modelled as follows

where the binary information source represents the system up to the output of the Huffman encoder. The discrete channel models the binary on-off keyed Transmitter/Receiver (with
\(x_1 = 1\) and \(x_2 = 0\) and the additive white Gaussian noisy channel with noise having a double-sided power spectral density of \(10^{-3}\) W/Hz.

(a) Estimate the bit-error probability of the system. 5%
(b) Find the information rate and the bit data rate (symbol rate) at the channel input. 10%
(c) Estimate the data point \((\text{EUE,BUE})\), where EUE denotes the energy utilization efficiency and BUE represents the bandwidth utilization efficiency of the system. 10%
(d) Estimate the information point \((\text{EUE,BUE})\), where EUE denotes the information energy utilization efficiency and BUE represents the information bandwidth utilization efficiency of the system. 15%
(e) Is the system a ‘realizable’ communication system? 5%
(f) What is the signal-to-noise ratio SNR, at the receiver’s input? 5%