PROBLEM SHEET - FT Tables Refresher

EE3-27: Principles of Classical and Modern Radar

Contents

1 Evaluating Functions of δ-lines

Evaluate:

(a) ${}_{-\infty}\int^{\infty}(t^4 - 3t + 1).\delta(t-2)\ .dt$ (10%)

(b) ${}_{-\infty}\int^{\infty}\left\{(\cos(4\pi t) * \delta(t+\frac{1}{4})\right\}.\delta(t-\frac{1}{8}).dt$ (10%)

(c) $\int\limits_{-\infty}^{\infty}(t^3 - 3t^2 - 11).\delta(t-1)\ .dt$ (5%)

(d) $\int\limits_{-\infty}^{\infty}\left\{(\sin(4\pi t) * \delta(t+\frac{1}{4})\right\}.\delta(t-\frac{1}{4}).dt$ (5%)

(e) $\int\limits_{-\infty}^{\infty}(t^3 - 2t^2 + 1).\delta(t-2)\ .dt$ (5%)

(f) $\int\limits_{-\infty}^{\infty}\left\{(\cos(2\pi t) * \delta(t-\frac{1}{4})\right\}.\delta(t-\frac{1}{12}).dt$ (5%)

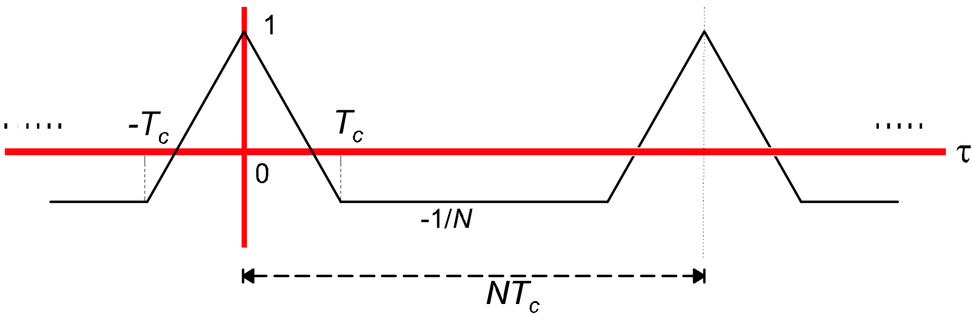
(g) $h(3)$ where $h(t) = \left(t.\text{rect}\left\{\frac{t}{8}\right\}\right) * \delta(t+3)$ (5%)

(h) $h(3)$ where $h(t) = \left(t.\text{rect}\left\{\frac{t}{8T}\right\}\right) * \delta(t-2)$ (10%)

(i) $h(3.5)$ where $h(t) = \left(t.\text{rect}\left\{\frac{t}{8T}\right\}\right) * \delta(t-3)$ (10%)

2 Auto-Correlation Function and PSD(f)

The waveform below shows the autocorrelation function $R_{bb}(\tau)$ of what is called in communications a pseudo-random (PN) signal $b(t)$.



(a) Write a mathematical expression, using Woodward's notation, to describe the above autocorrelation function. (15%)

(b) Find the power spectral density $\text{PSD}_b(f)$ of $b(t)$. (20%)

3 Transfer Function and PSD(f)

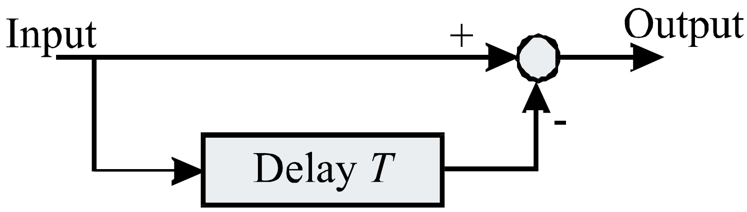
At the input of a filter there is white Gaussian noise of power spectral density $\text{PSD}_{n_i}(f) = \frac{3}{2}10^{-6}$. If the transfer function of the filter is

$$H(f) = \Lambda\left\{\frac{f}{10^6}\right\}\exp\left(-j\phi(f)\right)$$

calculate the power of the signal at the output of the filter. (10%)

4 Differential Circuit

For the following differential circuit



find:

(a) the impulse response and (5%)

(b) frequency response (5%)

5 Impulse Response and PSD(f)

Consider the filter with impulse response

$$h(t) = \text{sinc}^2 \left\{10^6(t-3)\right\}$$

and assume that the input signal $n_i(t)$ is white Gaussian noise with double-sided power spectral density $\text{PSD}_{n_i}(f) = 1.5 \times 10^{-6}$ W/Hz.

For the signal $n(t)$ at the output of the filter

(a) find and plot its power spectral density $\text{PSD}_n(f)$; (10%)

(b) calculate its power P_n (5%)

6 PSD(f) at Bandpass Filter's Output

Consider a bandpass filter with impulse response

$$h(t) = 8 \times 10^3 \text{sinc}\left\{4 \times 10^3 t\right\} . \cos(2\pi 10^4 t)$$

and assume that at the input of this filter there is white Gaussian noise $n_i(t)$ of power spectral density $\text{PSD}_{n_i}(f) = 10^{-6}$.

For the signal $n(t)$ at the output of the filter

(a) find and plot its power spectral density $\text{PSD}_n(f)$; (10%)

(b) calculate its power P_n (5%)

END