Study Group

Professor A. Manikas

Imperial College London

Comms-1

FM

PROBLEM SHEET : 8

Note Title

FM : $k_f \rightarrow \frac{Hz}{volts}$

PM : $k_p \rightarrow \frac{rads}{volts}$

1. Consider the FM signal

$$S(t) = 10\cos[2\pi F_c t + 2\pi k_f \int_{-\infty}^{t} g(u)du]$$

where $k_f = 10\pi$. The message $g(t)$ is given by

$$g(t) = \sum_{n=0}^{2} m_n(t) = m_0(t) + \underbrace{m_1(t)}{} + m_2(t)$$

with

$$\boxed{m_n(t) = \frac{2^n}{\pi}\mathrm{sinc}(t)\cos(2nt).}$$

(a) Sketch and dimension the Fourier transform of $m_1(t)$.

(b) Sketch and dimension the Fourier transform of $g(t)$.

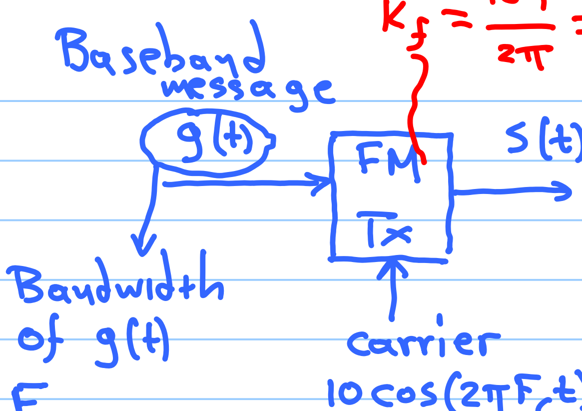
(c) Using Carson's rule, determine the bandwidth of $S(t)$.

[$75/\pi$Hz]

(d) Assume now that $g(t) = Ae^{-10t}u(t)$. Using Carson's rule, the bandwidth of $S(t)$ is 50.4 Hz. Find the amplitude A of $g(t)$. Select the bandwidth, B, of the baseband message $g(t)$ so that it contains 95% of the signal energy.

[$A = 1$]

$k_f = \frac{10\pi}{2\pi} = 5\,\frac{Hz}{V}$

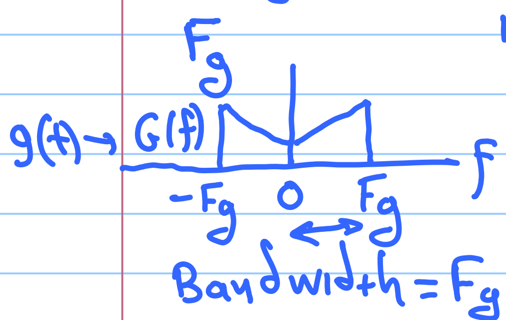


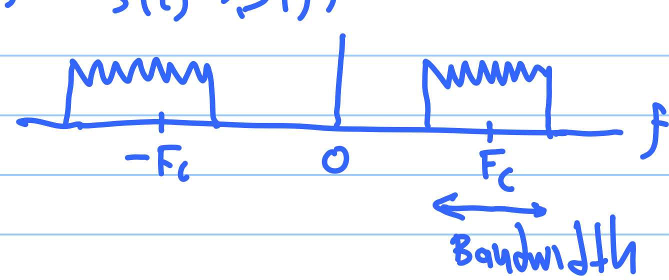
$S(t) = 10\cos(2\pi F_c t + 2\pi k_f \int_{-\infty}^{t} g(u)du)$

$\downarrow$

$\boxed{\text{Bandpass}}$

$s(t) \rightarrow S(f)$





a) $m_1(t) \xrightarrow{FT} M_1(f) = ?$

$g(t) = \underbrace{m_0(t)}_{} + m_1(t) + m_2(t)$

given: $\boxed{m_n = \frac{2^n}{\pi}\text{sinc}(t)\cos(2nt)}$

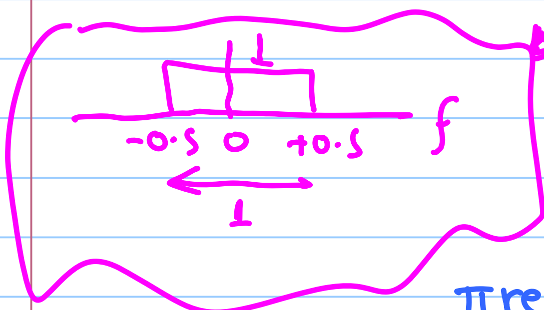
$m_0(t) = \frac{2^0}{\pi}\text{sinc}(t)\cos(\underbrace{2\times 0\times t}_{0}) = \frac{1}{\pi}\text{sinc}(t) =$

$= \frac{1}{\pi}\text{sinc}\left(\frac{\pi t}{\pi}\right)$

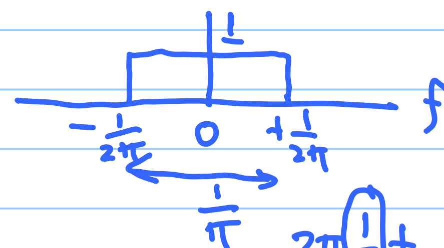
$\text{sinc}(\pi t) = \frac{\sin(\pi t)}{\pi t}$

$M_0(f) = \frac{1}{\pi}\,\pi\,\text{rect}(f\pi)$

remember rect(f)



$= \text{rect}\,\frac{f}{1/\pi}$



$\pi\,\text{rect}\left(\frac{f}{1/\pi}\right)$

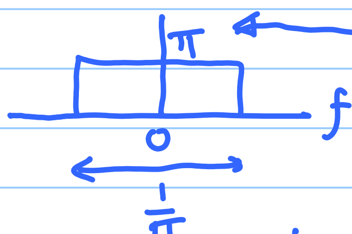
FT

$m_1(t) = \frac{2^1}{\pi}\,\text{sinc}(t)\cdot\cos(2\times 1\times t) = \frac{2}{\pi}\,\text{sinc}(t)\cdot\cos(2t)$

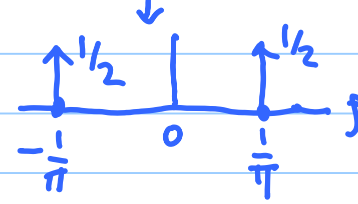
$2\pi\frac{1}{\pi}t$

Frequ.

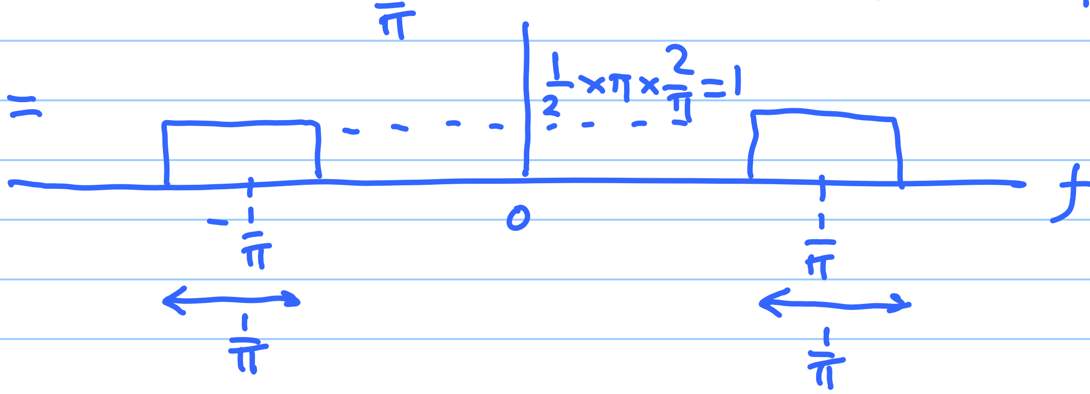
$M_1(f) = \frac{2}{\pi} \times$



$\circledast$ conv.

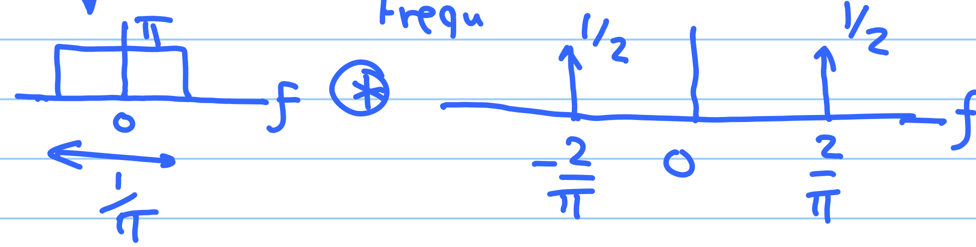


$M_1(f) =$

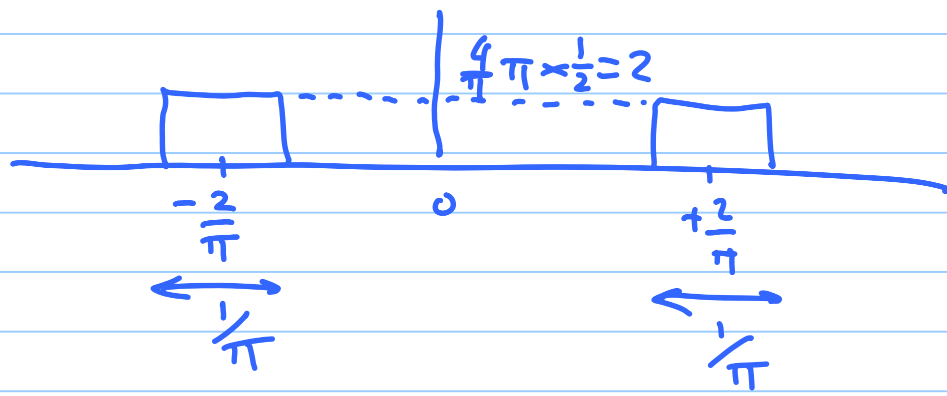


$$m_2(t) = \frac{2^2}{\pi}\,\operatorname{sinc}(t)\,\cos(2\times 2\times 1)$$

$$= \frac{4}{\pi}\,\operatorname{sinc}(t)\,.\,\cos\Big(2\pi\,\underbrace{\frac{2}{\pi}}_{\text{Frequ}}\,t\Big)$$

$M_2(f) = \frac{4}{\pi} \times$ 

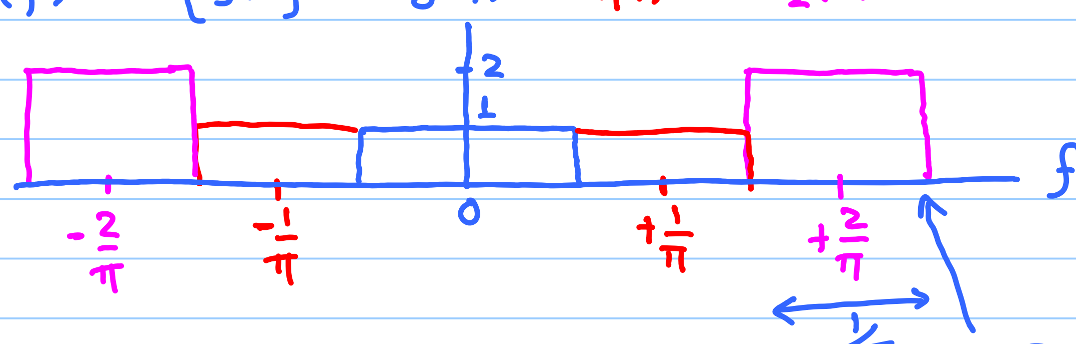
$\Downarrow$

$M_2(f) =$ 

b) $g(t) = m_0(t) + m_1(t) + m_2(t)$

$\downarrow$ FT

$$G(f) = FT\{g(t)\} = M_0(f) + M_1(f) + M_2(f)$$

$=$ 

$F_g = \frac{2}{\pi} + \frac{1}{2\pi}$

c) Carson's rule : $B = 2(F_D + F_g)$

$F_D \downarrow$

$k_f \cdot \max(g(t))$

$k_f \downarrow\ 5\ \frac{Hz}{V}$

$\max(g(t)) = \frac{1}{\pi} + \frac{2}{\pi} + \frac{2^2}{\pi} = \frac{7}{\pi}\ V$

$$\Rightarrow \quad B = 2\Big(\underbrace{\overbrace{K_f \max(g(t))}^{F_D}}_{5 \times \frac{7}{\pi}} + \underbrace{F_g}_{\frac{2}{\pi}+\frac{1}{2\pi}}\Big) = \frac{75}{\pi}$$

d) $g(t) = A\exp(-10t)\cdot u(t)$

$$G(f) = FT\{g(t)\} = \frac{A}{10 + j2\pi f}$$

$$\text{Power of } g(t) = P_g = \int_{-\infty}^{+\infty} \underbrace{PSD_g(f)}_{=|G(f)|^2} \cdot df = \frac{A^2}{20}$$

$$P_g \times \frac{95}{100} = \int_{-B}^{+B} \underbrace{|G(f)|^2}\, df$$

$$\Rightarrow \frac{\cancel{A^2}}{20} \times \frac{95}{100} = \int_{-B}^{+B} \frac{\left(\frac{\cancel{A}}{2\pi}\right)^2}{f^2 + \left(\frac{10}{2\pi}\right)^2}\, df \;\cdots\Rightarrow \boxed{B = 20.2\ Hz}$$

Above the follow expression may be used

$$\int_{-\infty}^{\infty} \frac{dx}{x^2 + a^2} = \frac{1}{a}\tan^{-1}\frac{x}{a}.$$