Study Group

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Comms-1

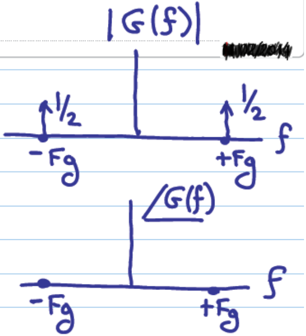
* FT of sinewaves
* PSD(f) of signals
* Transfer function of systems

SINEWAVES

PS4[1]

Note Title [redacted]

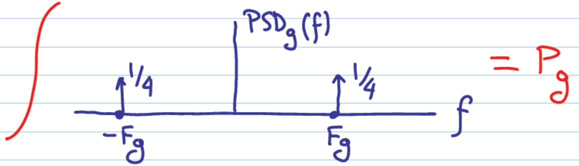
$g(t) = \cos(2\pi F_g t) \xrightarrow{\text{FS or FT}}$



ie $|G(f)| = \frac{1}{2}\delta(f+F_g) + \frac{1}{2}\delta(f-F_g)$

$\angle G(f) = 0$

$$PSD_g(f) = G(f)\cdot G(f)^* = |G(f)|^2 =$$
$$= \frac{1}{4}\delta(f+F_g) + \frac{1}{4}\delta(f-F_g)$$



ie $P_g = \int PSD_g(f)\,df = \frac{1}{4} + \frac{1}{4} = \frac{1}{2}$

indeed $P_g = \underbrace{\int \frac{1}{4}\delta(f+F_g)\,dg}_{\frac{1}{4}} + \underbrace{\int \frac{1}{4}\delta(f-F_g)\,dg}_{\frac{1}{4}} = \frac{1}{2}$

otherwise: $P_g = \overline{g^2(t)} = \overline{\cos^2(2\pi F_g t)} = 1/2$

or , using auto-correlation

$$R_{gg}(\tau) = \overline{g(t)\cdot g(t-\tau)} =$$

$$= \overline{\cos(2\pi F_g t)\cos(2\pi F_g(t-\tau))}$$

$$= \overline{\cos(2\pi F_g t)\cdot\left[\cos(2\pi F_g t)\cos(2\pi F_g \tau)+\sin(2\pi F_g t)\sin(2\pi F_g \tau)\right]}$$

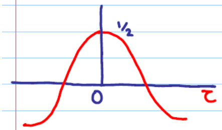
$$= \overline{\cos^2(2\pi F_g t)\cos(2\pi F_g \tau)} + \overline{\cos(2\pi F_g t)\sin(2\pi F_g t)\cdot\sin(2\pi F_g \tau)}$$

$$= \underbrace{\overline{\cos^2(2\pi F_g t)}}_{1/2}\cos(2\pi F_g \tau) + \underbrace{\overline{\cos(2\pi F_g t)\cdot\sin(2\pi F_g t)}}_{0}\cdot\sin(2\pi F_g \tau)$$

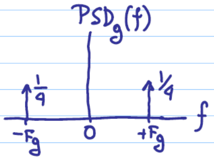
$$= \frac{1}{2}\cos(2\pi F_g \tau)$$

ie

$$R_{gg}(\tau) = \frac{1}{2}\cos(2\pi F_g \tau)$$



F.T. $\longrightarrow$



PS4.[2]

SYSTEM

convolution

input

$g(t)$ → $h(t) \xrightarrow{FT} H(f)$ → output

impulse response

transfer function

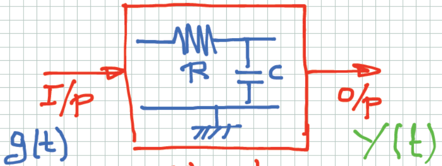
$$y(t) = g(t) \circledast h(t)$$

$$g(t) \downarrow G(f) \downarrow PSD_g(f)$$

$$Y(f) = G(f) \cdot H(f)$$

$$PSD_y(f) = PSD_g(f) \cdot |H(f)|^2$$

<u>note</u> : $H(f) =$ complex → $|H(f)|$, $\angle H(f)$



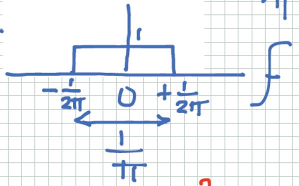
$$H(f)=\frac{1}{1+j2\pi f}$$

$$\Downarrow$$

$$|H(f)|^2=\frac{1}{1+(2\pi f)^2}$$

$$PSD_g(f)=\text{rect}(\pi f)=\text{rect}\left(\frac{f}{1/\pi}\right)$$

$=$



$$PSD_y(f)=PSD_g(f)\cdot|H(f)|^2$$

2(a)

$$P_y = \int_{-\infty}^{\infty} PSD_y(f)\,df$$

$$= \int_{-\infty}^{\infty} PSD_g(f) \cdot |H(f)|^2\,df$$

$$= \int_{-\infty}^{\infty} \mathrm{rect}(\pi f)\,\frac{1}{1+(2\pi f)^2}\,df$$

$$= \int_{-\frac{1}{2\pi}}^{\frac{1}{2\pi}} 1 \cdot \frac{1}{1+(2\pi f)^2}\,df$$

$$= \frac{1}{2\pi}\tan^{-1}(2\pi f)\Big|_{-\frac{1}{2\pi}}^{\frac{1}{2\pi}}$$

$$= \frac{1}{2\pi}\tan^{-1}1 - \frac{1}{2\pi}\tan^{-1}(-1)$$

$$= \frac{1}{2\pi}\cdot\frac{\pi}{4} + \frac{1}{2\pi}\cdot\frac{\pi}{4}$$

$$= \frac{1}{4}$$

2(b)

$$P_y = \int_{-\infty}^{\infty} PSD_y(f)\cdot df$$

$$= \int_{-\infty}^{\infty} PSD_g(f)\,|H(f)|^2\, df$$

$$= \int_{-\infty}^{\infty} \frac{1}{2\pi}\delta\left(f-\frac{1}{2\pi}\right)\underbrace{|H(f)|^2}_{\boxed{\frac{1}{1+(2\pi f)^2}}} df + \int_{-\infty}^{\infty} \frac{1}{2\pi}\delta\left(f+\frac{1}{2\pi}\right)\underbrace{|H(f)|^2}_{\boxed{\frac{1}{1+(2\pi f)^2}}} df$$

Note:

$$PSD_g(\omega) = \delta(\omega-1) + \delta(\omega+1)$$
$$\Downarrow$$
$$PSD_g(f) = \frac{1}{2\pi}\delta\left(f-\frac{1}{2\pi}\right) + \frac{1}{2\pi}\delta\left(f+\frac{1}{2\pi}\right)$$

$$= \frac{1}{2\pi}\,\frac{1}{1+\left(2\pi\frac{1}{2\pi}\right)^2} + \frac{1}{2\pi}\,\frac{1}{1+\left(-2\pi\frac{1}{2\pi}\right)^2}$$

$$= \frac{1}{2\pi}\,\frac{1}{2} + \frac{1}{2\pi}\,\frac{1}{2}$$

$$= \frac{1}{2\pi}$$