Study Group

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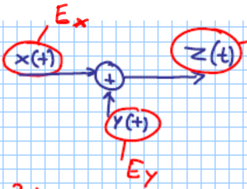
Comms-1

Cross – Correlation
&
Cross - Correlation Coeff.

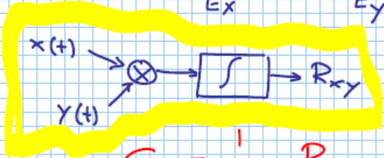
$z(t) = x(t) + y(t)$ ie



$E_z = \text{function}(E_x, E_y)$

$$E_z = \int z^2(t)\,dt = \int (x(t)+y(t))^2\,dt$$

$$= \underbrace{\int x^2(t)\,dt}_{E_x} + \underbrace{\int y^2(t)\,dt}_{E_y} + 2\underbrace{\int x(t)\,y(t)\,dt}_{\text{this is the cross-correlation}}$$



$\Longleftrightarrow$

$$R_{xy} = \int x(t)\,y(t)\,dt$$

$$= \underbrace{C_{xy}}_{\text{cross correlation coeff.}} \cdot \sqrt{E_x} \cdot \sqrt{E_y}$$

$$C_{xy} = \frac{1}{\sqrt{E_x}\sqrt{E_y}} R_{xy}$$

where $-1 \leq C_{xy} \leq +1$

Q1

N.B.: $R_{xy}=0 \Rightarrow x(t) \perp y(t)$ or $x(t), y(t)$ have zero similarity

(or $C_{xy}=0$)

orthogonal

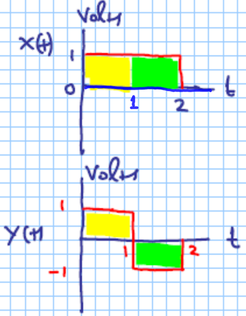
$R_{xy}\neq 0 \Rightarrow x(t)$ & $y(t)$ have some similarity $(x(t) \not\perp y(t))$

$C_{xy}=1 \Rightarrow x(t)$ & $y(t)$ are identical

$C_{xy}=-1 \Rightarrow x(t)$ & $y(t) \Rightarrow x(t)=-y(t)$

ie identical but opposite

PS2.Q1



$$E_x=\int x^2(t)\,dt=\int_0^2 1\,dt=2$$

$$E_y=\int y^2(t)\,dt=\int_0^1 1^2\,dt+\int_1^2 (-1)^2\,dt=2$$

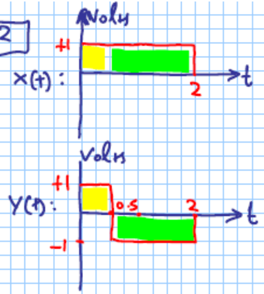
$$E_z=\int z^2(t)\,dt=E_x+E_y+2\underbrace{R_{xy}}=4$$

$$R_{xy}=\int_0^2 x(t)y(t)\,dt=$$

$$=\int_0^1 1\times 1\,dt+\int_1^2 1\times(-1)\,dt=0$$

$C_{xy}=0$

PS2.Q2



$E_x = 2$

$E_y = 2$

$$R_{xy} = \int x(t)y(t)dt = \underbrace{\int_0^{0.5} x(t)y(t)dt}_{=0.5} + \underbrace{\int_{0.5}^{2} x(t)y(t)\,dt}_{=-1.5}$$

$$= 0.5 + (-1.5) = -1$$

$$\boxed{C_{xy} = -0.5}$$

$$E_z = E_x + E_y + 2R_{xy}$$

$$= 2 + 2 + 2\times(-1) = 2$$