Problem Sheets: Communication Systems

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DS-CDMA

1. A recorded conversation is to be transmitted by a QPSK Direct Sequence Spread Spectrum System (DS/SSS). Assuming the spectrum of the speech waveform is bandlimited to 4 kHz, and that a 128-level quantizer is used:

   (a) find the chip rate required to obtain a processing gain of 20 dB, 10%

   (b) given that the sequence length is to be greater than 5 hours, find the number of shift register stages required. 10%

   Solution

   (a) $F_s = 2 \times 4K = 8KHz$
   $\gamma = \log_2 Q = \log_2 128 = 7$
   $r_b = \gamma F_s = 7 \times 8K = 56Kbits/s$
   $T_{cs} = 2T_b = 2\frac{1}{r_b} = 2\frac{1}{56K} = 3.5714 \times 10^{-5}$
   PG$= 20dB \Rightarrow 10\log_{10}\frac{T_{cs}}{T_c} = 20 \Rightarrow 100 = \frac{T_{cs}}{T_c} \Rightarrow T_c = \frac{T_{cs}}{100} = 0.35714 \times 10^{-6}$
   i.e. chip rate$= r_c = \frac{1}{T_c} = \frac{1}{0.35714\times10^{-6}} = 2.8$Mchips/s

   (b) $N_c T_c = 5\text{hours} \Rightarrow N_c \geqq \frac{5\text{hours}}{T_c} = \frac{5\times3600}{T_c} = \frac{5\times3600}{0.35714\times10^{-6}} = 0.0504 \times 10^{12}$
   $\Rightarrow N_c = 2^m - 1 \Rightarrow 2^m = N_c + 1 \Rightarrow m = \log_2(N_c+1) = \log_2(0.0504\times10^{12}+1) = 35.553$
   i.e. $m = 36$

2. Consider a DS-BPSK CDMA systems where the received powers from all users are equal to 10^{-2} (a perfectly power controlled system). The system operates in the presence of additive white Gaussian noise of double sided power spectral density 0.5×10^{-11} while the processing gain of the system is 400. If the bit rate for each user is 25 kbits/sec and the Signal-to-Noise-plus-Interference ratio at the output of the j^{th} receiver is equal to 14, how many users are supported by the system? 50%

   Solution

   $P = 10^{-2}$

   $\text{SNIR}_{out} = 14$

   $r_{cs} = 25 \Rightarrow T_{cs} = \frac{1}{25k}$

   PG=400$\Rightarrow$PG$= \frac{B_{ss}}{B} = \frac{T_{cs}}{T_c} \Rightarrow T_c = \frac{T_{cs}}{\text{PG}} = 10^{-7}$

   $B_{ss} = \frac{1}{T_c} = 10\text{MHz}$

   $\text{SNIR}_{out} = 2\text{EUE}_{equ} = 2\frac{E_b}{N_0+N_j} = 2\frac{PT_{cs}}{N_0+(K-1)\frac{P}{B_{ss}}}$

$\Rightarrow N_0 + (K-1)\frac{P}{B_{ss}} = \frac{2PT_{cs}}{\text{SNIR}_{out}}$

$\Rightarrow K = \left(\frac{2PT_{cs}}{\text{SNIR}_{out}} - N_0\right)\frac{B_{ss}}{P} + 1 \simeq 58$ users

3. Consider a digital cellular DS-BPSK CDMA communication system which employs three directional antennas each having 120° beamwidth, thereby dividing each cell into 3 sectors. The system can support up to 201 users/subscribers and operates with a data bit-rate of 500 kbits/sec in the presence of additive white Gaussian noise of double-sided power spectral density 10^{-9}. With a bit-error-probability for each user of 3×10^{-5}, a power equal to 10 mWatts, and a voice activity factor $\alpha = 0.375$, find:

   (a) the average energy per bit E_b, 5%

   (b) the equivalent EUE (EUE_{equ}), 5%

   (c) the processing gain (PG) of the system. 10%

   Solution

   (a) $P = 10mW$
   $r_b = 500\,\text{kbits/ sec} \Rightarrow T_{cs} = \frac{1}{500}$ msec

   (b) $K = 201$ users
   $N_0 = 2\times10^{-9}$
   $p_e = 3\times10^{-5}$
   $a = 0.375$
   $s = 1/3$
   $E_b = PT_{cs} = 10\times10^{-3}\times\frac{1}{500}\times10^{-3} = 2\times10^{-8}$
   $p_e = \text{T}\{\sqrt{2\,\text{EUE}_{equ}}\} \Rightarrow 3\times10^{-5} = \text{T}\{\sqrt{2\,\text{EUE}_{equ}}\}$
   $\Rightarrow$ (using "tail graph" supplied)
   $4 = \sqrt{2\,\text{EUE}_{equ}} \Rightarrow \text{EUE}_{equ} = 8$

   (c) However, $\text{EUE}_{equ} = \frac{E_b}{N_0+N_j}$
   where $E_b = PT_{cs}$ and $N_j = \frac{(K-1).P.a.s}{B_{ss}} = \frac{(K-1).P.a.s}{\text{PG}/T_{cs}}$
   Therefore, $\text{EUE}_{equ} = \frac{PT_{cs}}{N_0 + \frac{(K-1).P.a.s}{\text{PG}/T_{cs}}} \Rightarrow \ldots\ldots\ldots \Rightarrow \text{PG} = \frac{(K-1).P.a.s.T_{cs}}{\frac{PT_{cs}}{\text{EUE}_{equ}} - N_0}$
   $\Rightarrow \ldots \Rightarrow \text{PG} = 1000$

4. Consider a DS-BPSK CDMA system of 256 users where each user has a protection probability equal to 10^{-2} and an Anti-jam Margin of 30 dB. Each user employs a feedback shift register of 21 stages, whose feedback connections are described by a primitive polynomial. The system is perfectly power controlled and the received power from each user is equal to $P = 0.1915$W operating in the presence of additive white Gaussian noise of double sided power spectral density 0.5×10^{-6} Watts/Hz. Find:

   (a) the average energy per bit E_b and 20%

   (b) the PN-code rate. 10%

   Solution

   (a) $K = 256$
   AJM=30dB $\log_{10}\text{EUE}_{equ} - 10\log_{10}\text{EUE}_{PR} = 30$
   $\Rightarrow \frac{\text{EUE}_{equ}}{\text{EUE}_{PR}} = 10^3$ (1)
   $p_{e,PR} = 10^{-2}$
   $m = 21 \Rightarrow N_c = 2^m - 1 \Rightarrow N_c = 2^{21} - 1 = 2.0972\times10^6$
   $P = 0.1915$
   $N_0 = 10^{-6}$

$p_{e,PR} = \mathrm{T}\{\sqrt{2\ \mathrm{EUE}_{PR}}\} \Rightarrow 10^{-2} = \mathrm{T}\{\sqrt{2\ \mathrm{EUE}_{PR}}\}$

using tail function graph we have

$\sqrt{2\ \mathrm{EUE}_{PR}} = 2.3(\text{or } 2.3263) \Rightarrow \mathrm{EUE}_{PR} = 2.645 \text{ (or } 2.7058)$ (2)

$(1) \bigwedge (2) \Rightarrow \mathrm{EUE}_{equ} = 2645 \text{ (or } 2705.8)$

$\mathrm{EUE}_{equ} = \frac{E_b}{N_0+N_j} = 2645 \text{ (or } 2705.8)$

$\Rightarrow \quad E_b = \mathrm{EUE}_{equ}(N_0 + \frac{(K-1).E_b}{N_c})$

$E_b\left(1 - \frac{\mathrm{EUE}_{equ}(K-1)}{N_c}\right) = \mathrm{EUE}_{equ}\ N_0$

$E_b = \frac{\mathrm{EUE}_{equ}\ N_0}{1-\frac{\mathrm{EUE}_{equ}(K-1)}{N_c}} = \frac{2705.8\times10^{-6}}{1-\frac{2705.8\times(256-1)}{2.0972\times10^6}} = 4.032\,5 \times 10^{-3}$

(b) $T_{cs} = \frac{E_b}{P} = \frac{4.032\,5\times10^{-3}}{0.1915} = 21.05\,7 \times 10^{-3} = 21.057ms$

$N_c = \frac{T_{cs}}{T_c} \Rightarrow T_c = \frac{T_{cs}}{N_c} = \frac{21.056\times10^{-3}}{2.0972\times10^6} = 10.04 \times 10^{-9} = 10.04\text{ns}$

PN-code-rate $= \frac{1}{T_c} = \frac{1}{10.04\times10^{-9}} = 99.602 \times 10^6 = \boxed{99.602\text{Mchips/s}}$

5. Consider a digital cellular DS-QPSK CDMA communication system with a Gray encoder/decode which employs three directional antennas each having 120° beamwidth, thereby dividing each cell into 3 sectors. The system operates with a data bit-rate 25 kbits/sec. in the presence of additive white Gaussian noise of double-sided power spectral density 10^{-9}, while the processing gain of the system is 400. With a desired bit-error-probability for each user 3×10^{-5}, a power equal to 5 mWatts, and a voice activity factor $\alpha = 0.375$, how many users/subscribers can be supported by the system? 30%

Solution

$P = 5mW$

$r_b = 25kbits/sec \Rightarrow T_{cs} = 2T_b = 2\frac{1}{r_b} = 2\frac{1}{25\times10^3} = \frac{1}{12500} = 8 \times 10^{-5}$

$\frac{N_0}{2} = 10^{-9} \Rightarrow N_0 = 2 \times 10^{-9}$

PG$= N_c = 400 \Rightarrow 400 = \frac{T_{cs}}{T_c} \Rightarrow T_c = \frac{8\times10^{-5}}{400} = 2 \times 10^{-7}$

$p_e = 3 \times 10^{-5}$

$a = 0.375$

$s = 3$

$p_e = \mathrm{T}\{\sqrt{2\mathrm{EUE}_{equ}}\} \Rightarrow 3\times10^{-5} = \mathrm{T}\{\sqrt{2\mathrm{EUE}_{equ}}\} \overset{\text{(using the tail function graph)}}{\Rightarrow} 4 = \sqrt{\mathrm{EUE}_{equ}}$

$\Rightarrow \mathrm{EUE}_{equ} = \frac{16}{2} = 8 \Rightarrow$

$$\left.\begin{array}{r} \frac{E_b}{N_0+N_j} = 8 \\ E_b = \frac{PT_{cs}}{2} = \frac{5\times10^{-3}\times8\times10^{-5}}{2} = 2 \times 10^{-7} \\ N_j = \frac{(K-1)Pas}{B_{ss}} \end{array}\right\} \Rightarrow \frac{E_b}{N_0+(K-1)PasT_c} = 8$$

$\Rightarrow K = (\frac{PT_{cs}/2}{8} - N_0)\cdot\frac{1}{PasT_c} + 1$

$\Rightarrow K = (\frac{5\times10^{-3}\times8\times10^{-5}}{2\times8} - 2 \times 10^{-9})\frac{1}{5\times10^{-3}\times0.375\times1/3\times2\times10^{-7}} + 1 = 185$

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