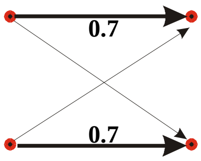
3 Topic: Communication Channels

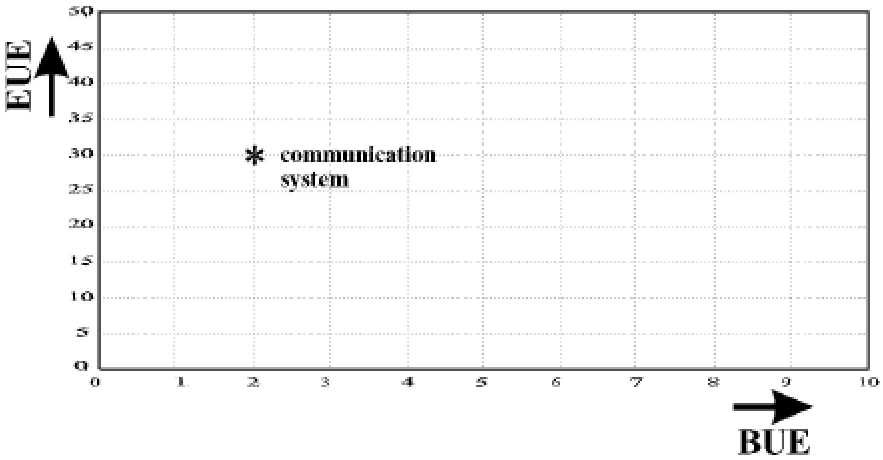
12. If one binary source and two binary channels are connected in cascade as shown below

where both channels have the following forward transition probability diagram



find the bit-error-rate p_e at the output of the second channel. 10%

13. A digital communication system, operating at 100 bits/sec in the presence of additive white Gaussian noise of power spectral density $\text{PSD}_n(f) = \frac{N_0}{2}$, is represented in the energy utilization efficiency (EUE) - bandwidth utilization efficiency (BUE) plane, as follows:



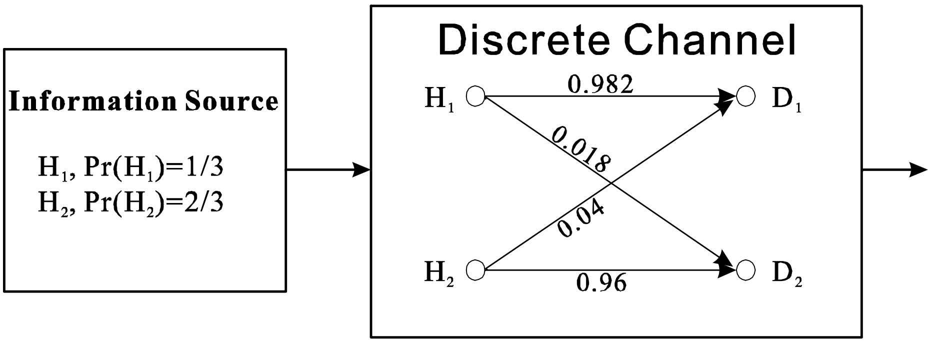
What is the capacity C of the channel in bits/sec? 20%

14. A digital communication system having an energy utilisation efficiency (EUE) equal to 30 operates in the presence of additive white Gaussian noise of double-sided power spectral density $\text{PSD}_n(f)= 0.5 \times 10^{-6}$ W/Hz. If the channel capacity C is 16 kbits/s and the channel bandwidth B is 4 kHz, estimate

    (a) the bit rate r_b 10%

    (b) the noise power at the channel output 10%

15. A discrete channel is modelled as follows:



Estimate:

(a) The probability of error at the output of the channel — 5%

(b) The amount of information delivered at the output of the channel — 10%

16. Consider a binary Communication System that uses the following two equally probable energy signals:

$$0 \quad \mapsto \quad s_0(t) = 2\Lambda\left\{\frac{t}{10\mu s}\right\}$$
$$1 \quad \mapsto \quad s_1(t) = -2\Lambda\left\{\frac{t}{10\mu s}\right\}$$

The channel is assumed additive white Gaussian noise of double-sided power spectral density $\mathrm{PSD}_n(f) = 10^{-6}\,\mathrm{W/Hz}$. Find:

(a) the bandwith B of the channel; — 5%

(b) the channel symbol rate r_{cs} (baud rate) & data bit rate; — 5%

(c) the Energy Utilisation Efficiency (EUE); — 10%

(d) the channel capacity C in bits/sec. — 10%

17. Consider a binary Communication System that operates with a bit rate 100kbits/sec and uses the following two equally probable energy signals:

$$0 \quad \mapsto \quad s_0(t) = 3\left(\Lambda\left\{\frac{t}{5\mu s}\right\} + \mathrm{rect}\left\{\frac{t}{10\mu s}\right\}\right)$$
$$1 \quad \mapsto \quad s_1(t) = -3\left(\Lambda\left\{\frac{t}{5\mu s}\right\} + \mathrm{rect}\left\{\frac{t}{10\mu s}\right\}\right)$$

The channel is assumed additive white Gaussian noise of double-sided power spectral density $\mathrm{PSD}_n(f) = 0.5 \times 10^{-6}\,\mathrm{W/Hz}$. Find:

(a) the bandwith B of the channel; — 5%

(b) the channel symbol rate r_{cs} (baud rate); — 5%

(c) the Energy Utilisation Efficiency (EUE); — 20%

(d) the channel capacity C in bits/sec. — 15%

18. Consider a binary digital communication system in which a binary sequence is transmitted as a signal $s(t)$ with a one being sent as $6\Lambda\left\{\frac{t}{T_{cs}/2}\right\}$ and a zero being sent as $-6\Lambda\left\{\frac{t}{T_{cs}/2}\right\}$. The source at the input to the system provides a binary sequence of ones and zeros, with the number of ones being twice the number of zeros. The transmitted signal is corrupted by channel noise $n(t)$ of bandwidth B and has an amplitude probability density function described by the following expression:

$$\text{pdf}_\text{n}(n) = \frac{1}{6}.\text{rect}\left\{\frac{n}{6}\right\}$$

Find a bound on the ratio C/B 20%

where C denotes the capacity of the channel in bits/s.

19. Consider a binary digital communication system in which the transmitted signal is corrupted by channel noise of bandwidth B having an amplitude probability density function described by the following expression:

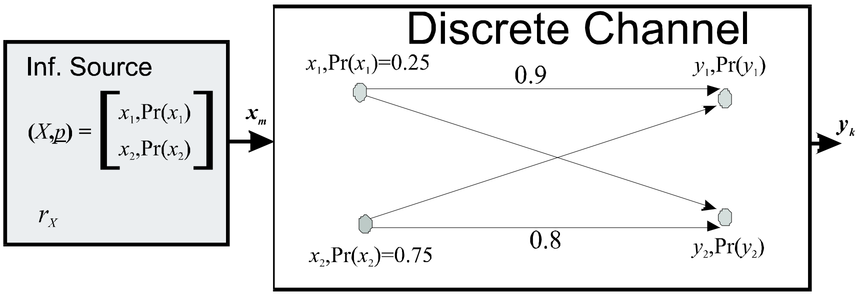
$$\text{pdf}_\text{n}(n) = \frac{1}{6}.\text{rect}\left\{\frac{n}{6}\right\}$$

If the power of the received signal is 12W then

(a) find the entropy power of the noise; 10%

(b) find an upper and a lower bound on the ratio C/B where C denotes the capacity of the communication channel. 10%

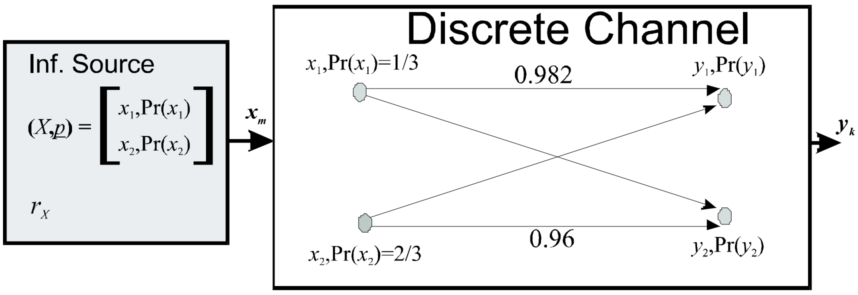
20. A discrete channel is modelled as follows:Estimate:



(a) The probability of error at the output of the channel 5%

(b) The amount of information delivered at the output of the channel 15%

21. A discrete channel is modelled as follows:



Estimate:

(a) The probability of error at the output of the channel 5%

(b) The amount of information delivered at the output of the channel 15%

22. A signal $g(t)$ bandlimited to 4kHz is sampled at the Nyquist rate and is fed through a 2-level quantizer. A Huffman encoder is used to encode triples of successive output quantization levels as follows:

symbols	probs	Huffman
$m_1 m_1 m_1$	27/64	1
$m_1 m_1 m_2$	9/64	001
$m_1 m_2 m_1$	9/64	010
$m_2 m_1 m_1$	9/64	011
$m_1 m_2 m_2$	3/64	00000
$m_2 m_1 m_2$	3/64	00001
$m_2 m_2 m_1$	3/64	00010
$m_2 m_2 m_2$	1/64	00011

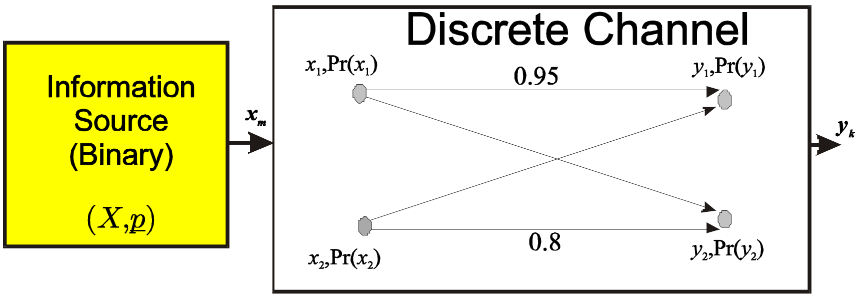
while the binary sequence at the output of the Huffman encoder is fed to a Binary on-off Keyed Communication System which employs the following two energy signals of duration T_{cs}

$$\begin{aligned} s_0(t) &= 0 \\ s_1(t) &= \sqrt{\frac{3}{8}}\boldsymbol{\Lambda}\left(\frac{t}{0.5\,T_{cs}}\right) \end{aligned}$$

The transmitted signals are corrupted by additive white Gaussian channel noise having a double-sided power spectral density of 10^{-3} W/Hz. The figure below shows a modelling of the whole system where the output of the Huffman encoder is modelled as the output of a binary discrete information source $(X, \underline{p})$ with

$$\begin{aligned} X &= \{x_1 = 1, x_2 = 0\}, \\ \underline{p} &= [\Pr(x_1), \Pr(x_2)]^T \end{aligned}$$

while the binary on-off Keyed system is modelled as a discrete channel as shown below.



(a) Find the entropy of the information source $(X, \underline{p})$, the information rate and the bit data rate (symbol rate) at the channel input. 15%

(b) Estimate the bit-error probability of the system. 10%

(c) Estimate the energy utilization efficiency (EUE) and bandwidth utilization efficiency (BUE) using the bit data rate as well as the information rate. 15%

(d) Represent the communication system, as a point on the (EUE,BUE) parameter plane. In this plane show also the locus of the system properly labelled. 10%

(e) Is the system a 'realizable' communication system? 5%

(f) What is the signal-to-noise ratio, SNR_{in}, at the receiver's input? 5%

23. A signal $g(t)$ having the pdf shown in Figure 1 is bandlimited to 4 kHz. The signal is sampled at the Nyquist rate and fed through a 2-level quantizer. The transfer function of the quantizer is shown in Figure 2.

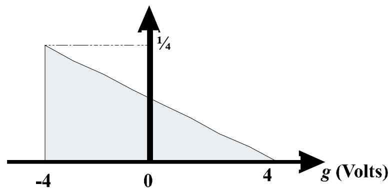


Figure-1

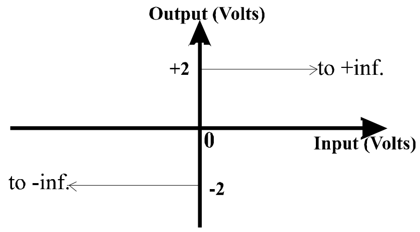
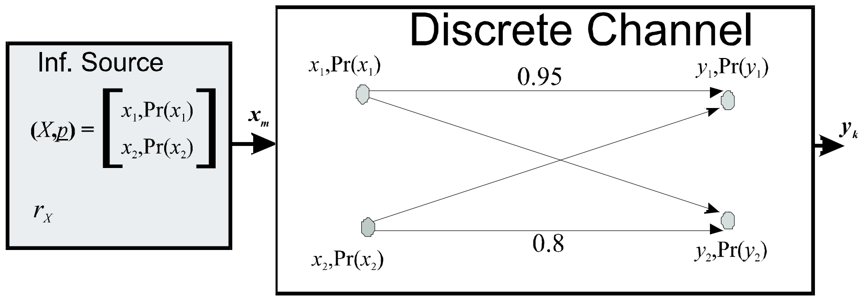


Figure-2

A Huffman encoder is used to encode triples of successive output quantization levels while the binary sequence at the output of the Huffman encoder is fed to a Binary on-off Keyed Communication System which employs the following two energy signals

$$s_1(t) = 0; s_2(t) = 0.5 \cos\left(2\pi \frac{5}{T_{cs}} t\right) ; \text{with } 0 < t < T_{cs}$$

The whole system is modelled as follows



where the binary information source represents the system up to the output of the Huffman encoder. The discrete channel models the binary on-off keyed Transmitter/Receiver (with $x_1 = 1$ and $x_2 = 0$) and the additive white Gaussian noisy channel with noise having a double-sided power spectral density of 10^{-3} W/Hz.

(a) Estimate the bit-error probability of the system. 5%

(b) Find the information rate and the bit data rate (symbol rate) at the channel input. 10%

(c) Estimate the data point (EUE,BUE), where EUE denotes the energy utilization efficiency and BUE represents the bandwidth utilization efficiency of the system. 10%

(d) Estimate the information point (EUE,BUE), where EUE denotes the information energy utilization efficiency and BUE represents the information bandwidth utilization efficiency of the system. 15%

(e) Is the system a 'realizable' communication system? 5%

(f) What is the signal-to-noise ratio SNR, at the receiver's input? 5%