Principles of Code Division Multiple Access (CDMA)

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EE303 - Communication Systems
An Overview of Fundamentals
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(a) SSS:

\[ b(t) \]

\[ \text{SS Mod.} \]

\[ \text{PN-generator} \]

\[ \{\alpha[k]\} \]

\[ r_c = 1/T_c \text{ bauds} \]

\[ n_i(t) \]

\[ j(t) \]

\[ B_{ss} \]

\[ \text{Digital Demod.} \]

\[ (M, T_{ca}) \]

\[ \text{bits} \]

\[ \text{carrier} \exp(j2\pi F_c t) \]

\[ \beta \]

\[ \hat{b}(t) \]

(b) CDMA \((K \text{ users})\):

\[ b(t) \]

\[ \text{SS Mod.} \]

\[ \text{PN-generator} \]

\[ \{\alpha[k]\} \]

\[ r_c = 1/T_c \text{ bauds} \]

\[ n_i(t) \]

\[ j(t) \]

\[ B_{ss} \]

\[ \text{Digital Demod.} \]

\[ (M, T_{ca}) \]

\[ \text{bits} \]

\[ \text{carrier} \exp(j2\pi F_c t) \]

\[ \beta \]

\[ \hat{b}(t) \]
The PN signal $b(t)$ is a function of a PN sequence of ±1’s $\{\alpha[n]\}$

- The sequences $\{\alpha[n]\}$ must agreed upon in advance by Tx and Rx and they have status of password.

- This implies that:
  - knowledge of $\{\alpha[n]\}$ $\Rightarrow$ demodulation = possible
  - without knowledge of $\{\alpha[n]\}$ $\Rightarrow$ demod. = very difficult

- If $\{\alpha[n]\}$ (i.e. “password”) is purely random, with no mathematical structure, then
  - without knowledge of $\{\alpha[n]\}$ $\Rightarrow$ demodulation = impossible

- However all practical random sequences have some periodic structure. This means:
  \[ \alpha[n] = \alpha[n + N_c] \] (1)
  where $N_c$ = period of sequence
  i.e. pseudo-random sequence (PN-sequence)
Remember

- **DS-SSS** (Examples: DS-BPSK, DS-QPSK):
  \[ b(t) = \sum_n \alpha[n].c(t - nT_c) \]  \hspace{1cm} (2)

  where \( \{\alpha[n]\} \) is a sequence of \( \pm 1 \)'s;
  \( c(t) \) is an energy signal of duration \( T_c \)

- **FH-SSS** (Examples: FH-FSK)
  \[ b(t) = \sum_n \exp \{j(2\pi k[n] F_1 t + \phi[n])\} . c(t - nT_c) \]  \hspace{1cm} (3)

  where \( \{k[n]\} \) is a sequence of integers such that

  \[ \{\alpha[n]\} \leftrightarrow \{k[n]\} \]  \hspace{1cm} (4)

  and \( \{\alpha[n]\} \) is a sequence of \( \pm 1 \)'s;
  \( c(t) \) is an energy signal of duration \( T_c \)
Basics of CDMA

**BLOCK DIAGRAM**

- \( K = \text{number of users} \)
- \( K-1 = \text{multiple access interference} \)
Example: DS-BPSK CDMA System

\[ P_e = \prod \left\{ \sqrt{\text{SNIR}_{\text{out}}} \right\} \]

Multiple Access Channel

SNIRin

\[ r(t) = \sum_{i=1}^{nT_e} \frac{b_i(t - \tau_i)}{(n-1)T_e} G_i + \text{noise} \]

SISO = Scalar-Input Scalar-Output Channel
SISO Multipath Channel

- SISO Multipath channel of the $i$-th user

In the absence of multipaths the above diagram has only $\tau_{i1}$ and $\beta_{i1}$ terms.

For the simplicity we will drop the second subscript and we will use $\tau_i$ and $\beta_i$, and thus the BPSK/DS-CDMA in the absence of multipaths may be represented as follows:
Basic Properties of CDMA Systems

- CDMA is one of the applications of spread spectrum communications which is used in civilian, commercial and military communication.
- Two systems: DS-CDMA (i.e. averaging system) and FH-CDMA (i.e. avoidance system).
  In this course only DS-CDMA will be considered.
- Assign a specific PN-code to each user
- PN-code (having the status of ‘password’) acts like a ‘channel’
- DS-CDMA: two main cases
  - PN-signal period \( = N_c \, T_c = T_{cs} \) (known as ‘short codes’ CDMA)
  - PN-signal period \( = N_c \, T_c \gg T_{cs} \) (known as ‘long codes’ CDMA)
- DS-CDMA: two main types
  1. synchronous DS-CDMA
     \[
     \begin{align*}
     & i\text{-th user} \\
     & j\text{-th user}
     \end{align*}
     \]
  2. non-synchronous DS-CDMA
     \[
     \begin{align*}
     & i\text{-th user} \\
     & j\text{-th user}
     \end{align*}
     \]
**DS-CDMA: Synchronization**

- The Rx requires a replica of the PN code, with the correct clock phase, in order to despread the signal.
- Therefore, Rx = “synchronization circuits” + “demod. circuits”
- The process of synchronizing the receiver to the transmitter’s PN code consists of two stages:
  - Acquisition (coarse synchronization).
  - Tracking (fine synchronization).

Operation: acquisition; tracking + demodulation; loose tracking; acquisition; tracking+demodulation; ......etc...........
Mobile Cellular Systems: Conventional & CDMA

- A mobile cellular system consists of base stations, cells (a cell is the area serviced by a base station) and mobiles (subscribers). When a call originates, the base station negotiates with the mobile on various aspects (such as the channel used etc.), before establishing communications. After this, as the mobile moves from cell-to-cell, the service is handed (hand-off or handover) from one base station to another.

- Only one base station will service a mobile at any one time.

- Note:
  - base station to mobile is known as FORWARD LINK
  - mobile to base station is known as REVERSE LINK
- Type of channels:

<table>
<thead>
<tr>
<th>UPLINK</th>
<th>DOWNLINK</th>
</tr>
</thead>
<tbody>
<tr>
<td>Traffic Channel</td>
<td>Traffic Channel</td>
</tr>
<tr>
<td>Access Channel</td>
<td>Pilot Channel</td>
</tr>
<tr>
<td></td>
<td>Synchron. Channel</td>
</tr>
<tr>
<td></td>
<td>Paging Channel</td>
</tr>
</tbody>
</table>

- Frequency Division Duplex (FDD) and Time Division Duplex (TDD)
Channel Reuse and Reuse Distance

- There is interference from other cells sharing the same channels. The reuse distance $D$, in these systems, is determined by the worst case interference situation.

- Current cellular systems = FDMA/TDMA
  - Most of the current cellular systems, such as GSM, use frequency division multiplex - time division multiplex (FDM-TDM) technique to improve the system capacity. In these systems, each user is assigned one time-frequency slot.
    - When the system gets larger, slots $\neq$ unique for each and every user as this will limit the system capacity. Therefore these slots (time/frequency) have to be reused (reused in cells separated by $D$ (cells), which is the reuse distance of the system).
The system capacity could be increased by increasing the number of channels available in a single cell, i.e. reducing the reuse distance $D$. But this reduction is limited by the co-channel interference, (i.e. the interference from other cells sharing the same channels). The reuse distance $D$, in these systems, is determined by the worst case interference situation.
In a CDMA system, the available spectrum and time are not split into distinct slots. Instead the whole (available) spectrum is used by each user.

Since the same frequency channel could be used by all the users/subscribers, the reuse distance $D$ could be reduced to 1, i.e.

$$\text{if CDMA then } D = 1$$
Signal Overlay

- The spread spectrum signal, from a CDMA system, has a very low power spectral density and, therefore, a CDMA system can overlay on top of existing narrow-band mobile cellular systems (of the same frequency band).

- This is because the interference (due to CDMA signals), added to a narrow-band mobile system channel, is very low and, therefore, the presence of CDMA signal will hardly affect the performance of the narrow-band mobile system.

- The CDMA system, however, needs to perform some extra processing to reject the narrowband interference due to the presence of the narrow-band signals.

Comment:
The capacity and performance of a mobile cellular system could be significantly improved by using CDMA techniques. In the paper “On the Capacity of a Cellular CDMA”, IEEE Transactions on Vehicular Technology, Vol.40, 1991 (by Gilhousen et al) the improvement in the capacity is discussed and it is stated that “no other proposed scheme appears to even approach this (CDMA) performance”.

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Analysis of a Direct Sequence BPSK CDMA System

- objective: to relate the BER $p_e$ with the total number of users $K$ as well as with the $\text{EUE}_{equ}$ at the receiver.
  
i.e.
  \[ p_e = f\left\{ \text{EUE}_{equ}, K \right\} \]  

- Main Assumptions
  - single cell system of $K$ users,
  - $\nexists$ multipaths
  - PN code period $= N_c = PG$
  - System = perfectly power-controlled
    (all SS signals arrive at the receiver with the same power)
    - NB: power control can often be implemented in practice with great accuracy.
  - System = totally asynchronous (there is no common timing reference for the transmitters/users)
    - NB: This is actually an advantage of CDMA over other multiple access techniques, because all users can transmit independently and no signalling information is required.
Note that the carrier of \( i^{th} \) transmitter is \( \sqrt{P_i} \cdot \exp(j(2\pi F_c t + \psi_i)) \).
- The period of each user’s PN-sequence is selected as $N_c = \frac{T_{cs}}{T_c}$, and therefore there is one code period per data bit (or $N_c$ chips per bit). Thus, for the BPSK case, the processing gain $PG$ is:

$$PG = N_c = \frac{T_{cs}}{T_c}$$ (8)

- The transmitted signal $s_i(t)$ of the $i$-th user is therefore

$$s_i(t) = \sqrt{P_i}.m_i(t).b_i(t).\exp(j(2\pi F_c t + \psi_i))$$ (9)

where $F_c$ is assumed common for all carriers.
Since the transmitters are not time-synchronous, there is a different time delay $\tau_i$ for each signal $s_i(t)$ before it reaches the receiver, with $0 \leq \tau_i < T_{cs}$ for $i = 1, 2, 3, \ldots, K$. The carrier phases $\psi_i$ are also assumed different so that $0 \leq \psi_i < 2\pi$ for $i = 1, 2, 3, \ldots, K$. Thus, ignoring the band-pass filters at the transmitters and the receiver, the received signal $r(t)$ can be described as follows:

$$r(t) = \sum_{i=1}^{K} \beta_i \sqrt{P_i} \cdot m_i(t - \tau_i) \cdot b_i(t - \tau_i) \cdot \exp(j(2\pi F_c t + \phi_i)) + n(t)$$

where

$$\phi_i = \psi_i - 2\pi F_c \tau_i$$

with $n(t)$ denoting the additive white Gaussian channel noise of double sided power spectral density $N_0/2$.
If the transmitted signals are all synchronized, the delays $\tau_i, \forall i$, are neglected. Synchronizing all transmitted signals requires a common timing reference and compensation for transmission delays in various transmission paths. This complicates the system requirements and has no clear advantage.

The receiver has a local PN-signal generator as well as a carrier generator that generate exact replicas of those used in the transmitter. The receiver is in perfect synchronism with the transmitter by using acquisition and tracking techniques (i.e. $\tau_1, \phi_1$ are known).

Thus, without loss of generality, let us assume that $\tau_1 = 0, \phi_1 = 0$.

The output of the 1-st correlator (desired) at $t = T_{cs}$ is thus

$$G_1 = \frac{1}{T_{cs}} \int_{(n-1)T_{cs}}^{nT_{cs}} r(t) \cdot b_1(t) \cdot \exp(-j(2\pi F_c t)) dt$$  \hspace{1cm} (12)
**SNIR_out as a function of EUE, N_c and K**

- At the output of the correlator (i.e. $G_1$) we have:

  $$\text{SNIR}_{\text{out}} \approx 2.\text{EUE}_{\text{equ}} = 2 \frac{E_b}{N_j + N_0}$$  \hspace{1cm} (13)

- However,

  $$N_j = (K-1).\frac{P}{B_{ss}} = (K-1).P.T_c = \frac{(K-1).PT_{cs}}{N_c} = \frac{(K-1).E_b}{N_c}$$  \hspace{1cm} (14)

- Therefore

  $$\text{SNIR}_{\text{out}} \approx 2.\text{EUE}_{\text{equ}} = \frac{E_b}{\left(\frac{K-1}{N_c}\right) + N_0} = \frac{1}{\frac{K-1}{2N_c} + \frac{1}{2E_{\text{UE}}}}$$  \hspace{1cm} (15)
BER as a function of EUE, $N_c$ and $K$

We have seen that

$$SNIR_{out} \approx \left\{ \frac{K-1}{2N_c} + \frac{1}{2 \text{ EUE}} \right\}^{-1}$$  \hspace{1cm} (16)$$

However,

$$p_e = T \left\{ \sqrt{SNIR_{out}} \right\}$$  \hspace{1cm} (17)$$

which implies that

$$p_e = T \left\{ \sqrt{2 \text{ EUE}_{equ}} \right\} = T \left\{ \frac{1}{\sqrt{\frac{K-1}{2N_c} + \frac{1}{2 \text{ EUE}}} } \right\}$$  \hspace{1cm} (18)$$
BPSK Examples

BPSK Transmitter ($j^{th}$-user)

$w_j(t)$

$b_j(t)$

$w_j(t) \cdot b_j(t)$

$A_c \cdot \cos(2\pi f_c t)$

$s_j(t)$

Time (ms)
Analysis of a Direct Sequence BPSK CDMA System

BPSK Examples

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$S_j(t)$

$R(t)$

$B_j(t)$

$R(t) \cdot B_j(t)$

BPSK Receiver ($j^{th}$ user)

Input to integrator in the presence of noise
Analysis of a Direct Sequence BPSK CDMA System

BPSK Examples

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(a) 2 Users

(b) 10 Users

Input to integrator in the presence of noise and 2 users

BPSK Receiver ($j$th-user)

Input to integrator in the presence of noise and 10 users
In order to achieve the full benefits of using CDMA, the transmitted signal powers $P_{s_i}$ should be controlled in such a way that received signal power, from all the users at a cell, are the same. This makes power control a key feature of CDMA mobile systems.
Voice Activity Factor

- Human speech contains a lot of pauses where there is no data to transmit. Thus a speaker is active for about half the time due to listening and pauses in speech.

> The fraction of time that a speaker is active is known as the voice activity factor $a$

- Extensive studies have shown that $0.35 < a < 0.5$. A popular value used is $a = \frac{3}{8} = 0.375$

- The voice activity feature can be taken into account in a communication system by suppressing the transmission when voice is absent. Assuming that we have a scheme where the carrier is turned-off during the speech idle periods then a reduction in interference (by a factor of the voice activity) can be achieved.
Some Important CDMA System Components

- Implementation of voice activity:
  \[
  \begin{align*}
  \text{TDMA/FDMA:} & \quad \text{very difficult} \\
  \text{CDMA:} & \quad \text{very easy}
  \end{align*}
  \]

- For a large number of users the capacity increases by a factor \(1/a\).

- Therefore, using the voice activity monitoring approach the capacity and the performance of a CDMA system will be improved (this improvement cannot be obtained in FDMA/TDMA systems).

- In particular the power of a user’s signal at a specific time instant can be expressed \(1 \times P_{\text{user}}\) with probability \(a\) and \(0 \times P_{\text{user}}\) with probability \((1 - a)\).

- Using voice activity the performance can be improved even more.

\[
\begin{align*}
\text{BPSK} & : \quad \text{SNIR}_{\text{out}} = 2.\text{EUE}_{\text{equ}} = 2 \frac{E_b}{N_j + N_0} \\
\text{where} \quad N_j & = \frac{(K - 1).P_s.a}{B_{ss}}
\end{align*}
\]

\[
(19) \quad (20)
\]
Therefore we can model the “on-off” activity of each user a binomial distribution, which implies that the probability that \( k \) user (out of \( K \)) are active is given as follows:

\[
Pr( k \text{ users are active}) = \binom{K}{k} a^k (1 - a)^{K-k}
\]  

(21)

where \( K \) is the number of users per cell

- Note that as \( K \) =\uparrow\Rightarrow \text{spread of distribution} = \downarrow

- Set a threshold \( K_{th}, \epsilon_{th} \) such that:

\[
Pr( \text{number of active users} > K_{th}) < \epsilon_{th}
\]  

(22)
Sectorization

- It is used in TDMA/FDMA and CDMA systems
**Sectorization**

- Sectorization is achieved by using directional antennas instead of omnidirectional antennas.
- Each cell is divided to three sectors using three directional antennas each having $120^\circ$ beamwidth.
- Using sectorization the performance can be improved even more. (The expected value of the total interference is reduced by a factor of $s = 3$ wrt single omnidirectional antenna case)

$$\text{BPSK} : \quad \text{SNIR}_{\text{out}} = 2.\text{EUE}_{\text{equ}} = 2 \frac{E_b}{N_j + N_0}$$  \hspace{1cm} (23)

where $N_j = \frac{(K - 1).P_s}{B_{ss}}$  \hspace{1cm} (24)

In practice: $3 \text{ dB} < \text{SNIR}_{\text{out}} < 15 \text{ dB}$
A better approach is to use three linear antenna arrays (smart antennas)

antenna array  switched beam array system  adaptive array system