Spatiotemporal Arrayed MIMO Radar: Joint Doppler, Delay and DOA Estimation

Harry Commin, Student Member, IEEE and Athanassios Manikas*, Senior Member, IEEE

Abstract—In this paper, a new method is presented for the subspace-based joint estimation of Doppler frequencies, relative path delays and Directions of Arrival (DOAs) in Multiple Input Multiple Output (MIMO) radar.

An equivalent ‘virtual’ SIMO (Single Input Multiple Output) radar representation of the MIMO radar system is established which enables the analysis and direct exploitation of the full MIMO system geometry, even in the presence of Doppler and relative path delays.

By employing a special sequence of transmit waveforms, this virtual SIMO framework is incorporated into a novel space-time receiver architecture which performs subspace-based joint estimation of Doppler, delay and direction of arrival. Once these parameters have been estimated, estimation of complex path fading coefficients follows in a straightforward manner.

Index Terms—MIMO radar, array manifold, virtual array, array processing, differential geometry.

NOTATION

\( a, A \) Scalar

\( a, \hat{A} \) Column vector

\( \hat{A} \) Matrix

\( I_N \) \((N \times N)\) identity matrix

\( \mathbf{1}_N \) \((N \times 1)\) vector of ones

\( 0_N \) \((N \times 1)\) vector of zeros

\( A, \bar{A} \) For a given receiver parameter, \( A \), the equivalent transmitter parameter is denoted \( \bar{A} \)

\( (\cdot)^T \) Transpose

\( (\cdot)^H \) Conjugate transpose

\(|\cdot|\) Absolute value

\( ||\cdot|| \) Euclidean norm of vector

\([a]\) Smallest integer not less than \( a \)

\( a^b \) Element-by-element power

\( \exp(a) \) Element-by-element exponential

\( \text{vec}(\hat{A}) \) Vector formed by stacking columns of \( \hat{A} \)

\( \text{diag}\{a\} \) Diagonal matrix whose diagonal entries are \( a \)

\( \text{Tr}\{\cdot\} \) Matrix trace operator

\( \mathcal{E}\{\cdot\} \) Expectation operator

\( \otimes \) Kronecker product

\( \odot \) Hadamard (element-by-element) product

\( \mathbb{R} \) The set of real numbers

\( \mathbb{C} \) The set of complex numbers

I. INTRODUCTION

A N arrayed multiple input multiple output (MIMO) radar is a radar system which employs two antenna arrays: one to transmit and one to receive. For each of these arrays, the antenna elements are distributed in three-dimensional real space about a common reference point. Array signal processing in multiple-target MIMO radar is concerned with the task of exploiting the arrays’ geometries (antenna locations) in order to detect, resolve and estimate the various parameters of multiple radar targets [1, 2]. In this context, detection performance is defined as the capability of the system to correctly estimate the number of targets, \( K \). Then, resolution performance refers to the system’s ability to subsequently yield unique parameter estimates for those \( K \) targets. Estimation performance is then determined by the accuracy of those parameter estimates, following successful resolution.

In general, the directional detection, resolution and estimation performance of an array system is a function of the array aperture and number of sensors. In practice, these resources are limited and so the purpose of a `superresolution' direction-finding (DF) algorithm is to achieve high performance without increasing the size of the array. In particular, `superresolution' refers to the algorithm’s ability to achieve asymptotically infinite resolving capability as the number of data snapshots, \( L \), becomes large. Superresolution techniques have therefore been an important research topic in array signal processing for several decades [3].

Subspace-based parameter estimation algorithms (of which the MUSIC algorithm [4] is a characteristic example) have been shown to exhibit superresolution capability by seeking to locate intersections between an estimated signal subspace and the array’s ‘manifold’. Indeed, the key to designing and analysing an array system in general lies in understanding the array manifold, which is a geometric object that completely characterises the array. Specifically, the array manifold is defined as the locus of all the response vectors (manifold vectors) of the array over the feasible set of signal/target parameters. A branch of mathematics dedicated to the investigation of the properties of such geometric objects (curves, surfaces, etc.), is differential geometry. The profound and fundamental importance of the array manifold’s shape has been extensively investigated in the literature using differential geometry [5]. However, until now, these methods have been applied only to the receiver array of the array system. Therefore, in MIMO radar (where there also exists an arrayed transmitter), it has not previously been possible to fully characterise the whole transmit-receive system geometry within such a framework.
In this paper, an equivalent ‘virtual’ SIMO (Single Input Multiple Output) representation of the MIMO radar system is established which allows direct analysis of the full MIMO system geometry. The virtual array concept has already been introduced in the MIMO radar literature (see, for example, [6,7]). However, the existing theory was developed assuming the absence of relative path delays or Doppler and relies upon a matched filtering stage which fails when these effects are present. In this paper, a more general view of the virtual array is therefore adopted, which does not rely upon matched filtering and supports the inclusion of delays and Doppler effects in its modelling. Analytical expressions are then derived to describe explicitly the superior directional detection, resolution and estimation performance offered by the full MIMO configuration (compared to any method which only exploits the receiver array geometry).

The space-time receiver architecture (and related transmit waveform design) proposed in this paper exploits the superior DF performance afforded by the virtual SIMO array as part of a subspace-based joint Doppler, Delay and DoA estimation system. It is well known that a highly non-linear simultaneous three-parameter search (evaluated at all possible three-parameter combinations) would be too computationally complex for practical use. Therefore, by exploiting the specific structure of the transmitted waveforms, the method proposed in this paper instead partitions the estimation process into two consecutive stages (a single-parameter search followed by a two-parameter search). During this two-stage estimation procedure, complex path fading coefficients (which are a function of a given target’s radar cross section and range) are treated as nuisance parameters, but are shown to be readily estimated after Doppler, Delay and DoA estimates have been made available.

The remainder of the paper is organised as follows. In Section II, the MIMO radar system is modelled based on the array response vectors of the transmit and receive arrays. (Therefore, contrary to common practice in the MIMO wireless communications literature, the channel state information is not assumed to be known and must be estimated). In Section III, the equivalent virtual SIMO radar representation of the MIMO radar system is derived and analysed. The proposed receiver architecture is described in Section IV. Representative computer simulation results are presented in Section V, before the paper is concluded in Section VI.

II. MIMO RADAR SYSTEM MODEL

A. Received Array Signal Model

Consider the arrayed MIMO radar system of Figure 1, which employs an array of \( N \) transmitting antennas and an array of \( N \) receiving sensors. The two arrays are assumed to be collocated (i.e. they are located sufficiently close together in space that target bearings are the same for both arrays). The \( N \) elements of the transmit array are fed by the \( (N \times 1) \) vector of baseband transmit waveforms, \( \vec{m}(t) \), which are transmitted into the environment (see Point A in Figure 1) and assumed to propagate as plane waves. The proposed structure of \( \vec{m}(t) = \vec{m}(t)c(t) \) will be described fully in Section II-B.

The MIMO channel comprises \( K \) signal propagation paths, corresponding to the transmitted signal energy reflected back to the receiver via \( K \) radar targets. The signal return from the \( k^{th} \) target has complex path fading coefficient \( \beta_k \), while \( \tau_k \) models the lack of synchronisation between transmitter and receiver due to the different target ranges. Doppler frequency is denoted \( \mathcal{F}_k \) and is a known function of radial target velocity, \( v_k \):

\[
\mathcal{F}_k \triangleq -\frac{2\nu_k f_c}{c} \tag{1}
\]

where \( f_c \) denotes carrier frequency and \( c \) is the speed of light.

The \( (N \times 1) \) complex vector \( \vec{S}_k \triangleq \vec{S}(\theta_k, \phi_k) \) is the receiver array manifold vector (array response vector), which models the response of the receiver array to a plane-wave arrival from the direction parameterised by azimuth \( \theta_k \) and elevation \( \phi_k \):

\[
\vec{S}(\theta_k, \phi_k) \triangleq \exp \left( -j[k_{x}, k_{y}, k_{z}][\vec{S}(\theta_k, \phi_k)] \right) \tag{2}
\]

The receiver array’s sensor locations (array geometry) are represented by the \( (N \times 3) \) real matrix:

\[
[k_{x}, k_{y}, k_{z}] = [k_{1}, k_{2}, \ldots, k_{N}]^T \in \mathbb{R}^{N \times 3} \tag{3}
\]

Similarly, using (7) to denote all equivalent parameters associated with the transmit array, \( [\vec{x}_1, \vec{y}_1, \vec{z}_1] \), the transmit array manifold vector is denoted \( \vec{S}_k \triangleq \vec{S}(\theta_k, \phi_k) \).
In Equation 2, \( \mathbf{k}(\theta_k, \phi_k) \) is the wavenumber vector:

\[
\mathbf{k}(\theta_k, \phi_k) = \frac{2\pi}{\lambda} \begin{pmatrix} \cos(\theta_k) \cos(\phi_k) \\ \sin(\theta_k) \cos(\phi_k) \\ \sin(\phi_k) \end{pmatrix}
\]

(4)

where \( \lambda \) is the carrier signal wavelength and \( \mathbf{u}(\theta_k, \phi_k) \) is the \((3 \times 1)\) real unit vector pointing from \((\theta_k, \phi_k)\) towards the origin. (Equivalently, for the transmitter’s manifold vector, \( \mathbf{u}(\theta_k, \phi_k) = -\mathbf{u}(\theta_k, \phi_k) \) points from the origin to \((\theta_k, \phi_k)\).

Without loss of generality, phase origins of the transmit and receive arrays are defined at the centroids of the arrays. The \((N \times 1)\) baseband signal at the receiver array output (in the presence of noise) can therefore be modelled as follows:

\[
\mathbf{r}(t) = \sum_{k=1}^{K} \mathbf{R}_k \mathbf{S}_k \mathbf{m}(t) - \mathbf{H}(t) + \mathbf{n}(t)
\]

(5)

where the spatially and temporally white, zero-mean complex Gaussian additive noise is denoted by \( \mathbf{n}(t) \), with covariance matrix:

\[
\mathbb{E}_{nn} = \mathbb{E}\{\mathbf{n}(t)\mathbf{n}^H(t)\}
\]

\[
= \sigma_n^2 I_N
\]

(6)

where \( \sigma_n^2 \) is the unknown noise variance.

Without loss of generality, it will be assumed in this paper that all targets lie in the x-y plane (i.e. \( \phi_k = 0 \), for \( k = 1, 2, \ldots, K \)). Therefore, the unknown parameters of interest associated with the \( k^{th} \) target are Doppler \( F_k \), delay \( \tau_k \), DoA \( \theta_k \) and fading coefficient \( \beta_k \). In order to compare MIMO radar systems with different numbers of transmitting antennas, \( N \), in a fair manner, it is convenient to enforce a unity transmit power constraint:

\[
\text{Tr} \left\{ \mathbb{E}\left\{ \mathbf{m}(t)\mathbf{m}^H(t) \right\} \right\} = 1
\]

(7)

B. Proposed Transmit Waveform Design

As depicted in Figure 1, the transmitted baseband vector signal, \( \mathbf{m}(t) \), is formed using a repeating sequence, \( \mathbf{m}(t) \), of \( N \) orthogonal codes which are each spread using the same spreading sequence, \( c(t) \):

\[
\mathbf{m}(t) = \mathbf{m}(t)c(t)
\]

\[
= \sum_{n=1}^{N} [^n] \mathbf{a}[n] \mathbf{\alpha}[n]p((t - n - 1)2Nc - (i - 1)T_c)
\]

(8)

where \( \mathbf{a}[n] \in \mathbb{C}^{N \times 1} \) denotes the \( n^{th} \) vector of transmitted (discrete-time) symbols and \( p(t) \) is the chip-shaping waveform (herein assumed to be a simple rectangular chip waveform). Meanwhile, the spreading sequence, \( c(t) \), has a special discrete structure which is written as:

\[
\mathbf{c} = \left[ c(1), c(2), \ldots, c(2Nc) \right]^T
\]

\[
= \begin{pmatrix} \mathbf{b} \\ \mathbf{0}_{2Nc} \end{pmatrix} \in \mathbb{C}^{2Nc \times 1}
\]

(9)

III. THE VIRTUAL SIMO RADAR EQUIVALENT TO THE MIMO RADAR SYSTEM

A key challenge in MIMO radar is to determine how the transmit array geometry (described by \( \mathbf{a} \)) can be exploited effectively to enhance parameter estimation capability at the receiver. By contrast, the receiver array geometry is relatively straightforward to exploit in this sense, since each element of \( \mathbf{r}(t) \) is known to correspond to a specific receive antenna. Therefore, (assuming a calibrated array) signals arriving at the receive antennas have a known response as a function of \( \theta \), as described by \( \mathbf{S}(\theta) \). Consequently, a wide variety of parametric approaches can be applied to estimate \( \theta \), based upon this modelling.

One way of utilising transmit array geometry would therefore be to – in some sense – “virtually” transfer the transmit antennas across to the receiver. In this way, the output of a virtual SIMO array could be exploited at the MIMO radar receiver whose response is a function of both \( \mathbf{S}(\theta) \) and \( \mathbf{S}(\theta) \).

The structure of the virtual SIMO received signal can be derived by noting that the MIMO \( \mathbf{r}(t) \) can be rearranged as shown in Equation 13. The proof of this rearrangement is given in Appendix A. From \( \mathbf{z}_r(t) \) (in Equation 13), the structure of the equivalent virtual SIMO system can therefore
From Equations 15 and 18, it is evident that both the signal and noise powers at a given virtual sensor is a factor of $N$ smaller than in the MIMO radar system. This is to be expected, since the number of virtual sensors is greater by a factor of $N$ and the total signal and noise energies must be the same in both systems. Importantly, the signal to noise ratios are equal in both systems:

$$\text{SNR}_{v,k} = \text{SNR}_k$$  \hspace{1cm} (19)

Using Equation 14, the geometry of the virtual array can now be derived. Noting that $(\mathbf{S} \otimes \mathbf{S}) = (\mathbf{S} \otimes 1_N) \odot (1_N \otimes \mathbf{S})$, the virtual SIMO array sensor locations are given by:

$$\begin{pmatrix} r_{v,x}, r_{v,y}, r_{v,z} \end{pmatrix} \overset{\Delta}{=} \begin{pmatrix} \mathbf{r}_{v,x}, \mathbf{r}_{v,y}, \mathbf{r}_{v,z} \end{pmatrix} \odot 1_N + \begin{pmatrix} 1_N \otimes [r_{v,x}, r_{v,y}, r_{v,z}] \end{pmatrix}$$  \hspace{1cm} (20)

which can be viewed as a spatial convolution of the MIMO transmit and receive arrays.

Of course, in the actual observed MIMO signal vector, $\mathbf{x}(t)$, subvectors of $\mathbf{z}_v(t)$ are summed together according to $(1_N \otimes \mathbb{I}_N)$. Therefore, a key challenge in practice is to combine the information from multiple time instances in order to make $\mathbf{z}_v(t)$ available for processing. In the absence of relative path delays and Doppler, it has been shown that this goal can be achieved by applying a bank of matched filters at the front end of the receiver [6,7]. In this paper, a more general approach will be presented which allows the virtual array structure to be exploited, even when targets have different path delays and radial velocities.

A. Discrete-time Modelling

In order to employ digital signal processing at the MIMO radar receiver, the received signal, $\mathbf{z}(t)$, is first discretised (sampled). In particular, the $(N \times 2N_v)$ matrix of received data snapshots associated with the $n^{th}$ received symbol period (assuming a sampling period of $T_c$) can be written as:

$$\mathbf{X}[n] \triangleq [\mathbf{z}(0), \mathbf{z}(T_c), \ldots, \mathbf{z}((2N_v - 1)T_c)]$$

$$= \sum_{k=1}^{K} \beta_k \mathbf{S}_k \mathbf{S}_k^H \mathbf{a}[n] (\mathbf{y}_k \xi \odot \mathbf{f}_k)^T + N_v$$  \hspace{1cm} (21)

(or, equivalently, see Equation 22) where $N$ and $N_v$ are the matrices of additive noise and virtual noise snapshots,

$$\mathbf{X}[n] = (1_N^T \otimes \mathbb{I}_N) \left( \mathbf{a}[n] \otimes 1_N \right) \odot \sum_{k=1}^{K} \beta_k \exp(j2\pi \mathcal{F} t) \left( \mathbf{S}_k^H \otimes \mathbf{S}_k \right) (\mathbf{y}_k \xi \odot \mathbf{f}_k[n])^T + N_v$$  \hspace{1cm} (22)
respectively. Doppler effects are described by:

\[ \mathcal{F}_k[n] \triangleq \exp \left( j2\pi \mathcal{F}_k \begin{bmatrix} 0 \\ 1 \\ \vdots \\ 2N_c - 1 \end{bmatrix} T_c \right) \exp \left( 2\pi n \mathcal{F}_k 2N_c T_c \right) \]  

(23)

In Equation 21, discrete path delays have been defined as:

\[ t_k \triangleq \left\lfloor \frac{\tau_k}{T_c} \right\rfloor \]  

(24)

and \( J \) is the \((2N_c \times 2N_c)\) downshift matrix:

\[ J \triangleq \begin{bmatrix} 0 & 0 \\ 1 & 0 \\ \vdots & \vdots \\ 2N_c - 1 & 2N_c - 1 \end{bmatrix} \]  

(25)

It is worth noting that vectorising \( X[n] \) (row-wise) in Equation 22 gives rise to a virtual Doppler-STAR manifold vector, of similar structure as the receiver-only case in [9]. However, due to the way the three-parameter search (Doppler, delay and DoA) is partitioned in this paper, it is more useful here to use the structure shown in Equation 22.

B. Fundamental Performance Bounds of the Virtual SIMO Radar System

The fundamental detection and resolution performance bounds of a SIMO system were proven in [5] to be a function of array geometry and the finite sampling effect. In this respect, the array is fully characterised by the array manifold, respectively. The array is fully characterised by the array manifold, which is defined as the locus of the array manifold vector over the feasible set of signal/target parameters. Meanwhile, the finite sampling effect is represented by an ‘uncertainty hypersphere’ whose radius is given by:

\[ \sigma_e = \frac{1}{\sqrt{2(SNR \times L)}} \]  

(26)

Thus, by considering the circular approximation to the array manifold, it has been shown that the fundamental detection and resolution capabilities are characterised, respectively, by the following threshold separations:

\[ \Delta p_{\text{det}} = \frac{1}{\hat{s}(\hat{p})} \left( \sigma_{e_1} + \sigma_{e_2} \right) \]  

(27)

\[ \Delta p_{\text{res}} = \frac{1}{\hat{s}(\hat{p})} \sqrt{\frac{4}{\kappa_1^2(\hat{p}) - \frac{1}{N}}} \sqrt{\sigma_{e_1} + \sigma_{e_2}} \]  

(28)

where \( \Delta p \triangleq |p_2 - p_1| \) and \( \hat{p} \triangleq \frac{p_1 + p_2}{2} \) denote, respectively, the separation and midpoint between two closely-spaced targets with locations parameterised by \( p_1 \) and \( p_2 \) (for some generic directional parameter, \( p \), e.g. azimuth or elevation). Meanwhile, \( \hat{s}(\hat{p}) \) is the manifold’s rate of change of arc length and \( \kappa_1(p) \) denotes its principal curvature (where \( \kappa_1(p) \) also takes into account the inclination angle of the manifold). Similarly, the effect of geometry on fundamental estimation error performance is described by the Cramer-Rao Bound (CRB):

\[ \text{CRB}(p_1) = \frac{1}{(\text{SNR}_1 \times L)} \frac{2}{\hat{s}^2(p_1)\hat{s}^2(\hat{p}) \Delta p^2} \left( \kappa_1^2(\hat{p}) - \frac{1}{N} \right) \]  

(29)

In Appendix B, it is shown that the relevant geometric properties of the virtual manifold are given by:

\[ \hat{s}_v = \hat{s} \sqrt{N + N \frac{\hat{s}_v^2}{s^2}} \]  

(30)

\[ \hat{\kappa}_{1,v} = \frac{1}{\hat{s}_v^2} \sqrt{\left( \| \hat{A}_v \|_2^2 - 2 \hat{\kappa}_v \right) - \frac{\left( 1 + N \hat{s}_v^2 \right)^2}{s^2}} \]  

(31)

where a dot over a symbol denotes differentiation with respect to \( p \) and we have defined \( \hat{A}_v \triangleq -[\hat{L}_{v,x}, \hat{L}_{v,y}, \hat{L}_{v,z}] \hat{A}(p) \). Since \( \hat{\kappa}_{1,v} \) will tend to be smaller than \( \hat{\kappa}_1 \) (because the virtual manifold lies on a hypersphere of significantly larger radius), it is important to confirm that resolution/estimation performance is dominated by the improvement in \( \hat{s}_v \). Indeed, it is shown in Appendix B that:

\[ \hat{s}_v^4 \left( \hat{\kappa}_{1,v}^2 - \frac{1}{\hat{N}_v} \right) \geq N \hat{s}_v^4 \left( \hat{\kappa}_v - \frac{1}{N} - 1 \right) + 4\hat{s}_v^2 s^2 \]  

(32)

For illustrative purposes, an example showing \( \hat{s}_v(\theta) \) and \( \hat{s}(\theta) \) is given in Figure 3.

C. Parameter Identifiability

In the absence of Doppler or relative path delays, it was shown in [10] that the maximum possible number of targets detectable by a MIMO radar system (i.e. its ‘parameter identifiability’) is:

\[ K_{\max} \leq \frac{2NN}{3} \]  

(33)

However, it is evident from Equation 13 that, in the presence of relative path delays, the virtual signals arriving from different targets can be almost fully uncorrelated. This leads to a superior upper bound:

\[ K_{\max} \leq \frac{NN}{N} - 1 \]  

(34)
where equality will tend to be possible when all targets have different path delays.

The validity of this expression is demonstrated in Figure 5, where the MUSIC algorithm [4] is used at the receiver of a MIMO radar system with $\bar{N} = 4$, $N = 3$ (following methods described later, in Section IV). All $K = 11$ targets can be seen to be successfully resolved (whereas existing methods would fail for $K > 8$). Target azimuths: $[\theta_1, \theta_2, \ldots, \theta_{11}] = [37^\circ, 39^\circ, 60^\circ, 62^\circ, 63^\circ, 65^\circ, 66^\circ, 123^\circ, 125^\circ, 126^\circ, 127^\circ]$. It is also interesting to note that even very closely-spaced targets can be resolved due to the superresolution nature of the MUSIC algorithm.

It is important to note that these results refer to space-only considerations. As will be seen in Section V, by also exploiting the targets’ Doppler and temporal diversity, the number of detectable targets can vastly exceed the number of virtual antennas.

### IV. Joint Doppler, Delay and DoA Estimation

Having discussed the possible advantages offered by the virtual SIMO system in detail, a method is proposed in this section which exploits the virtual SIMO array as part of a joint Doppler, Delay and DoA estimation algorithm. To maintain focus on resolution and estimation, it will be assumed that the number of targets is known in advance, for example by using a detection algorithm such as Minimum Description Length (MDL) or Akaike’s Information Criterion (AIC) [11]. It is well known that a simultaneous three parameter search has a tendency to be prohibitively computationally complex. Therefore, the method proposed in this section instead decouples the three parameter search into two consecutive subspace-based parameter estimation operations:

1) (Single-parameter) delay estimation
2) (Two-parameter) joint DoA-Doppler estimation

It will be seen that partitioning the estimation process in this way is made possible due to the good autocorrelation properties of $c(t)$.

#### A. Preprocessing

The proposed joint Doppler-Delay-DoA estimation receiver architecture is depicted in Figure 4. At the front end, a bank of $N$ tapped delay lines (TDLs) is employed to store the target echoes associated with $D$ full periods of $\bar{m}(t)$ (Point “C” in Figure 4):

$$
X \triangleq [X[1], X[2], \ldots, X[D\bar{N}]] \in \mathbb{C}^{N \times 2D\bar{N}c}.
$$

Each time the contents of the TDLs are read out (at intervals of $2D\bar{N}cT_c$), the submatrices of $X$ are effectively stacked symbol upon symbol as follows (Point “D”):

$$
\tilde{X}_{\text{aug}} \triangleq \begin{bmatrix}
X[1] \\
X[2] \\
\vdots \\
X[D\bar{N}]
\end{bmatrix} \in \mathbb{C}^{D\bar{N}\times 2Nc}.
$$

36

$$
\tilde{X}_{\text{aug}} = \sum_{k=1}^{K} \beta_k h(\theta_k, \mathcal{F}_k) (\mathbf{j}^T \xi) + \mathcal{N}_{\text{aug}}
$$
where \( N_{\text{aug}} \) is the augmented matrix of stacked noise snapshots and it has been assumed that Doppler effects from chip to chip (corresponding to elements of \( \xi \)) can be neglected. In Equation 36, the \( (DNN \times 1) \) augmented virtual Space-Doppler response vector is related to the (space-only) virtual SIMO array manifold vector according to the following (Doppler-dependent) linear mapping:

\[
F(\theta_k, F_k) \triangleq \mathbf{A}_k \left( \mathbf{S}_k \otimes \mathbf{S}_k \right) \quad (37)
\]

Specifically, \( \mathbf{A}_k \), is defined as:

\[
\mathbf{A}_k \triangleq \left( \mathbf{F}_{\text{per}, k} \otimes \mathbf{I}_{NN} \right) \left( \text{diag} \left\{ \mathbf{F}_{\text{sym}, k} \right\} \mathbf{M}^T \otimes \mathbf{I}_N \right)
\]

(38)

where \( \mathbf{F}_{\text{per}} \) and \( \mathbf{F}_{\text{sym}} \), respectively, denote Doppler effects from period to period and from symbol to symbol:

\[
\mathbf{F}_{\text{per}, k} \triangleq \exp \left( j2\pi \mathbf{F}_k 2NN_c T_c \left[ 0, 1, \ldots, D - 1 \right]^T \right) \in \mathbb{C}^{D \times 1}
\]

(39)

\[
\mathbf{F}_{\text{sym}, k} \triangleq \exp \left( j2\pi \mathbf{F}_k 2NN_c T_c \left[ 0, 1, \ldots, N - 1 \right]^T \right) \in \mathbb{C}^{N \times 1}
\]

(40)

Therefore, \( F(\theta_k, F_k) \) can simply be viewed as the extended manifold vector obtained from \( \mathbf{S}_k(\theta_k) \) via the linear mapping \( \mathbf{A}_k \). In other words, the geometrical properties of the \( \theta \)-curves (which determine the system’s space-only capabilities) are characterised by the virtual SIMO array, while the space-Doppler response of \( h(\theta, F) \) as a whole is represented by a two-parameter manifold surface [12]. Note also that \( \mathbf{M}^T \) in Equation 38 is an isometric mapping and so has no effect on the intrinsic geometry of the array manifold [5]. Thus, since \( \mathbf{M} \) is a square matrix, it does not affect system performance (even for practical algorithms which introduce more uncertainty as the observation space grows larger [13]).

Forming \( \mathbf{X}_{\text{aug}} \) with the specific structure of Equation 36 provides the key to the decoupled estimation procedure. Clearly, the structure of \( h(\theta, F) \) suggests that joint DoA-Doppler estimation can be performed directly using \( \mathbf{X}_{\text{aug}} \). However, prior to doing this, the excellent autocorrelation properties of \( c(t) \) can first be exploited to separate the signal paths in time.

### B. Delay Estimation

The objective of the first estimation stage is to identify the set of \( K_r \leq K \) unique relative path delays present in the target environment. In the interest of clarity, we proceed by first transposing \( \mathbf{X}_{\text{aug}} \), to obtain:

\[
\mathbf{X}_{\text{aug}}^T = \sum_{k=1}^{K} \beta_k \left( \mathbf{J}_k \xi \right) h_k^T(\theta_k, F_k)
\]

(41)

where \( \mathbf{J}_k \xi \) can be viewed as the \( k^{th} \) temporal response vector’ and \( h_k^T(\theta_k, F_k) \) is the corresponding (sampled) ‘message’.

Now, the first step is to (for each discrete delay, \( l \), in the parameter search) construct the \( (2N_r \times N_c - 1) \) matrix, \( \mathbf{C}_l \), which comprises all feasible temporal response vectors except \( \mathbf{J}_\xi \):

\[
\mathbf{C}_l = [\mathbf{J}_0 \xi, \mathbf{J}_1 \xi, \ldots, \mathbf{J}^{l-1} \xi, \mathbf{J}^{l+1} \xi, \ldots, \mathbf{J}^{N_c-1} \xi]
\]

(42)

Then, following a similar approach to [14], \( \mathbf{X}_{\text{aug}}^T \) is simply left-multiplied by the following complement projection matrix:

\[
\mathbf{P}_{\mathbf{C}_l} = \mathbf{I}_{2N_r} - \mathbf{C}_l \left( \mathbf{C}_l^H \mathbf{C}_l \right)^{-1} \mathbf{C}_l^H
\]

(43)

This acts to completely eliminate all undesired contributions (i.e., those with delays not equal to \( l \)), while minimising attenuation of desired terms. Since \( \mathbf{J}_\xi^H \) is almost orthogonal to all columns of \( \mathbf{C}_l \), this attenuation will tend to be small and so decoupling the three-parameter search in this way does not significantly harm fundamental system performance. It is also important to note that the interference eliminated by \( \mathbf{P}_{\mathbf{C}_l} \) also includes any ‘coherent sources’ type cases which arise when two targets with different delays share the same DoA and radial velocity. Therefore, we simply proceed to calculate the sample covariance matrix:

\[
\mathbf{R}_l = \frac{1}{N_D} \mathbf{P}_l^H \mathbf{X}_{\text{aug}} \mathbf{X}_{\text{aug}}^H \mathbf{P}_l
\]

(44)

and then compute the principal eigenvector of \( \mathbf{R}_l \) (denoted \( \mathbf{\hat{\xi}}_l \)) in order to evaluate the following subspace-based cost function:

\[
\xi_{\text{delay}}(l) = \frac{2N_c}{(\mathbf{\hat{\xi}}_l^H \mathbf{P}_l^H \mathbf{P}_l \mathbf{\hat{\xi}}_l)}
\]

(45)

where \( \mathbf{P}_l^H \triangleq \mathbf{I}_{2N_r} - \mathbf{\xi}_l \mathbf{\xi}_l^H \) is the orthogonal projection onto the complement subspace to \( \mathbf{\xi}_l \). The estimated unique relative path delays, \( \hat{l} \triangleq \left[ \hat{l}_1, \hat{l}_2, \ldots, \hat{l}_{K_r} \right]^T \), are then obtained by locating the \( K_r \) largest values in \( \xi_{\text{delay}}(l) \) (Point “E”).

### C. Joint DoA-Doppler Estimation

Using the estimated unique relative path delays obtained using Equation 45, the corresponding estimated temporal response matrix is constructed as:

\[
\mathbf{\hat{C}} = [\mathbf{J}^0 \xi, \mathbf{J}^\hat{l}_1 \xi, \ldots, \mathbf{J}^\hat{l}_{K_r} \xi]
\]

(46)

Thus, \( \mathbf{X}_{\text{aug}} \) can now be transformed in such a way that the signal paths are separated into \( K_r \) columns, corresponding to the \( K_r \) unique path delays:

\[
\mathbf{Y} = \mathbf{X}_{\text{aug}} \left( \frac{\mathbf{\hat{C}}^H}{\mathbf{\hat{C}}} \right)^T
\]

(47)

Specifically, ignoring noise, the \( k^{th} \) column of \( \mathbf{Y} \) takes the form:

\[
\mathbf{y}_k = \mathbb{H}_k \beta_j_k
\]

(48)

where \( \mathbb{H}_k \in \mathbb{C}^{DN \times K_{ik}} \) denotes the matrix of all \( K_{ik} \) virtual Space-Doppler manifold vectors (see Equation 37) associated with the delay \( l_k \) and \( \beta_j_k \) is the vector of corresponding path fading coefficients.

Equation 48 describes a classic ‘coherent sources’ type scenario. Therefore, if any two (or more) targets share the same delay, additional steps must be taken to separate their paths (such steps are discussed in Section IV-D, below). However, for now, we proceed under the simplifying assumption that \( K_r = K \). Thus, joint DoA-Doppler estimation is achieved...
(for each of the \(K_r\) unique delays) by evaluating the following MUSIC-like subspace-based cost function:

\[
\xi_{\text{DoA-Dopp}}(\theta, \mathcal{F}) = \sum_{l=1}^{D} \left\{ \frac{\mathbb{H}^H(\theta, \mathcal{F}) \tilde{\mathbb{H}}(l, \mathcal{F}) \tilde{\mathbb{H}}^H(l, \mathcal{F})}{\| \tilde{\mathbb{H}}(l, \mathcal{F}) \|^2} \right\}
\]

where the columns of \(\mathbb{E}_{n,k}\) are the estimated noise eigenvectors obtained from the sample covariance matrix associated with \(\tilde{y}_k\) (or the space-Doppler ‘smoothed’ equivalent, as described in Section IV-D). The estimated DoAs and Doppler frequencies associated with the delay \(l_k\) are therefore obtained by locating the \(K_l\) largest maxima in \(\xi_{\text{DoA-Dopp}}(\theta, \mathcal{F})\). These estimates are denoted, respectively, as \(\hat{\theta}_{l_k}\) and \(\hat{\mathcal{F}}_{l_k}\) (Point “G”).

Although path fading coefficients, \(\beta\), were treated as nuisance parameters in the above two-stage parameter estimation process, they can now be estimated in a straightforward manner (Point “G”):

\[
\hat{\beta}_{l_k} = \hat{\mathbb{H}}^+ \tilde{y}_k
\]

where \(\hat{\mathbb{H}}\) is the estimated Space-Doppler channel response matrix, whose columns are constructed by inserting elements of \(\hat{\theta}_{l_k}\) and \(\hat{\mathcal{F}}_{l_k}\) (pairwise) into Equation 37.

D. ‘Decorrelating’ Coherent Sources

Whenever two or more targets share the same delay, Equation 48 describes a ‘coherent sources’ scenario. For certain specific receiver array geometries (in particular, uniform linear), Spatial Smoothing [15] can be overlaid directly to ‘decorrelate’ paths having different DoAs. Furthermore, if it is known a priori that radial velocities are sufficiently small such that \((\mathbb{I}_D \otimes \mathbf{M}^* \otimes \mathbf{1}_N) \mathbb{H}(\theta, \mathcal{F}) \approx \left( \mathcal{F}_{\text{per}} \otimes \hat{\mathbb{S}}^* \otimes \hat{\mathbb{S}} \right)\), and a very specific collinear transmit-receive array structure is used [16], then it is possible to apply spatial smoothing across the full extent of \(\hat{\mathbb{S}}^* \otimes \hat{\mathbb{S}}\).

Based on the same concept as Spatial Smoothing, a new decorrelating technique termed ‘Doppler Smoothing’ will now be described which exploits the ULA-like structure of \(\mathcal{F}_{\text{per}} \) in \(\mathbb{H}(\theta, \mathcal{F})\). As shown in Figure 6, Doppler smoothing requires the extraction of \(Q\) overlapping subvectors from \(\tilde{y}_k\). Each subvector is of length \(d N\), where \(d = D - Q + 1\). Denoting the \(q^{th}\) subvector of \(\tilde{y}_k\) as \(\tilde{y}_{k,q}\) and collecting these for all \(q = 1, 2, \ldots, Q\) yields:

\[
\tilde{y}_k = \begin{bmatrix} \tilde{y}_{k,1}; \tilde{y}_{k,2}; \cdots; \tilde{y}_{k,Q} \end{bmatrix}
\]

In an equivalent manner to Spatial Smoothing, this acts to ‘decorrelate’ paths at the expense of reducing the length of \(\mathcal{F}_{\text{per}}\) in \(\mathbb{H}(\theta, \mathcal{F})\) from \(D\) to \(d\). Note that, since Doppler Smoothing operates only on \(\mathcal{F}_{\text{per}}\), Spatial Smoothing can be overlaid independently on \(\tilde{y}_k\) by extracting the appropriate \(Q_{ss}\) subvectors from each column. The resulting \(Q_{ss}\) total subvectors are denoted by the matrix \(\tilde{y}_{\text{smooth},k}\), allowing us to finally compute the ‘smoothed’ covariance matrix:

\[
\tilde{\mathbb{R}}_{\text{smooth},k} = \frac{1}{Q_{ss}} \tilde{y}_{\text{smooth},k} \tilde{y}_{\text{smooth},k}^H
\]

**Algorithm Summary**

1) Preprocess:
   a) Sample \(\mathbf{x}(t)\) and collect snapshots in a bank of TDLs to form \(\mathbb{X}\) (Eq. 35).
   b) Read out contents of TDLs such that the \(\mathbb{N}\mathbb{D}\) submatrices comprising \(\mathbb{X}\) are stacked to form \(\mathbb{X}_{\text{aug}}\) (Eq. 36).

2) Estimate delays:
   a) for \(l = 1, 2, \ldots, N_c\)
      i) Apply complement projection \(\mathbb{P}_{\mathbb{C}}\) (Eq. 43) to \(\mathbb{X}_{\text{aug}}\) and compute \(\mathbb{R}_t\) (Eq. 44).
      ii) Calculate the principal eigenvector of \(\mathbb{R}_t\) and evaluate delay estimation spectrum \(\xi_{\text{delay}}(l)\) (Eq. 45).
   b) Obtain delay estimates at \(K_r\) largest values in \(\xi_{\text{delay}}(l)\).

3) Estimate DoA and Doppler:
   For \(l = l_1, l_2, \ldots, l_{K_r}\)
   a) Construct estimated temporal response matrix \(\hat{\mathbb{C}}\) (Eq. 46) and use it to compute \(\mathbb{Y}\) (Eq. 47).
   b) for \(k = 1, 2, \ldots, K_r\)
      i) Apply Doppler Smoothing and/or Spatial Smoothing and compute \(\tilde{\mathbb{R}}_{\text{smooth},k}\) (Eq. 52).
ii) Eigendecompose $\hat{\mathbf{R}}_{\text{smooth},k}$ and evaluate $\xi_{\text{DoA-Dopp}}(\theta, \mathcal{F})$ (Eq. 49).

iii) Obtain joint DoA-Doppler estimates at the $K_{th}$ highest spectral maxima in $\xi_{\text{DoA-Dopp}}(\theta, \mathcal{F})$.

4) Estimate path fading coefficients:
   a) Construct estimated Space-Doppler channel response matrix, $\hat{\mathbf{H}}$.
   b) Obtain fading coefficient estimates by inserting $\hat{\mathbf{H}}$ in Equation 50.

V. RESULTS

Two different simulation scenarios will be considered in this section. In the first, a planar arrayed MIMO configuration is shown to successfully resolve and estimate the DoAs, radial velocities, relative delays and fading coefficients of $K = 27$ radar targets. The targets are located across full 360° azimuth and ‘coherent sources’ problems are particularly abundant.

The second simulated environment was taken from [17] (wherein the Iterative Adaptive Approach (IAA) was applied to MIMO radar). Although the authors of [17] provide limited results for joint Doppler, delay and fading coefficient estimation, this is done using a simultaneous three-parameter iterative search (which is considered here to be prohibitively complex to compute). However, a stationary-targets environment is also simulated in [17], motivating a two-parameter search. Therefore, the proposed algorithm has been tested in the same environment in order to demonstrate that a similar estimation result is achieved (under a constant $(\text{SNR} \times L)$ constraint).

For all simulations, a carrier frequency of $f_c = 2GHz$ and chip period of $T_c = 0.8138\mu s$ are used and, in all trials, data is recorded for an observation interval of approximately 5.m.s. In each trial, we define the ‘nominal’ SNR as:

$$\text{SNR}_0 \triangleq \frac{\mathbf{H}^H \mathbf{R}_n \mathbf{H}}{\sigma_n^2}$$

from which it follows (from Equation 12) that the actual SNR of the $k^{th}$ target (at the input of the receiver array) is simply $\text{SNR}_k = |\beta_k|^2 \text{SNR}_0$. When Spatial or Doppler Smoothing are applied, the number of smoothing operations is chosen using the approximate optimal value suggested in [18].

A. Simulated Environment 1

The planar arrayed MIMO radar configuration is depicted in Figure 7. It employs an $N = 8$ element uniform linear receiver array, to which Spatial Smoothing is applied such that its effective aperture is reduced to 5 half-wavelengths. The transmit array is formed from an $\mathcal{N} = 5$ element half-wavelength-spaced uniform X-shaped array which has been ‘stretched’ by a factor of 5 parallel to the x-axis. As a result, the equivalent virtual SIMO array has no overlapping antennas (which share identical locations in space), while still ensuring that no array ambiguities associated with sparsely positioned antennas will occur.

The target environment comprises $K = 27$ moving and stationary targets, located across the full 360° azimuth. The nominal SNR is $\text{SNR}_0 = 10dB$ and fading coefficient magnitudes are selected independently from a uniform distribution on the interval $[\sqrt{0.1}, 1]$ (such that $0dB \leq \text{SNR}_k \leq 10dB$). All 27 targets span just two relative delays and numerous instances of each type of ‘coherent sources’ problems are present. Specifically: 14 targets share identical Doppler and delay with some other target(s); 13 share identical DoA and delay; and 8 share identical DoA and Doppler. Doppler Smoothing is employed with $D = 10$ and $d = 7$.

![First array configuration (in units of half-wavelengths).](image1)

Fig. 7: First array configuration (in units of half-wavelengths).

![Simulated Environment 1: Delay estimation spectrum, showing unique delays correctly estimated at 8$T_c$ and 9$T_c$.](image2)

Fig. 8: Simulated Environment 1: Delay estimation spectrum, showing unique delays correctly estimated at 8$T_c$ and 9$T_c$.

B. Simulated Environment 2

The stationary-target scenario simulated in this subsection was taken from [17]. Exact parameter values were not provided in [17], but the values used here are approximately the same. In this scenario, an $N = 5$ element receiver ULA and $\mathcal{N} = 5$ element transmitter ULA (stretched by a factor of 5 along the x-axis) are employed. The target environment comprises $K = 29$ stationary targets with azimuth values between $60^\circ$ and $120^\circ$. Fading coefficient magnitudes take values between approximately 0.01 and 1 (see Figure 13(a)).

Simulations in [17] were carried out with $\text{SNR}_0 = 20dB$ and $L = 64$. For lower SNRs, proportionally more snapshots are required for equivalent performance (see Equations 26 - 28), but increasing snapshots significantly increases computation time of the IAA method. Therefore, the purpose of the results presented here is to demonstrate the performance of the proposed algorithm with a significantly lower $\text{SNR}_0 = 0dB$. Accordingly, approximately 100 times more snapshots are
used (we set $L = 6350$, a $5.17\text{ms}$ observation interval), while all other system parameters are unchanged.

For this stationary-targets scenario, we take $D = 1$, and spatial smoothing is applied across the full aperture of the (uniform linear) virtual array. The parameter estimation results are shown in Figures 12 - 13, showing that accurate parameter estimates are obtained for 28 out of 29 of the targets (a similar result as achieved by IAA in [17]). It is worth noting that in cases where erroneous DoA-Doppler-delay estimates occur (e.g. when resolution fails), the corresponding estimated fading coefficient magnitude will tend to be close to zero, indicating robustness against false alarms.

VI. CONCLUSION

In this paper, a new method has been presented for joint Doppler, delay and DoA estimation in MIMO radar. By utilising an equivalent ‘virtual’ SIMO representation of the MIMO radar system, the fundamental detection and resolution capabilities of the proposed method are enhanced (compared to any approach which only exploits the receiver array geometry directly) even in the presence of Doppler and relative path delays. Since a full simultaneous three-parameter search is prohibitively complex to compute in practice, a two-stage estimation procedure is proposed in which delays are estimated first, before DoA and Doppler are estimated jointly.
(a) Delay estimation spectrum. Unique delays are correctly identified for the 16 different delays.

(b) DOA-delay estimation spectrum (where all target velocities are zero). 28 out of 29 targets are resolved and estimated with high accuracy (the only exception being a target with $|\beta|^2 = 10^{-4}$).

Fig. 12: Simulated Environment 2: Parameter estimation spectra for joint DOA and delay estimation (stationary targets).

(a) Magnitude estimates, with a green ‘X’ denoting true parameter values. When joint DOA-Doppler-delay estimation fails to resolve, fading coefficient magnitude estimates tend to be close to zero (see $k = 19$).

(b) Phase estimates, with a green ‘X’ denoting true parameter values. The spurious phase estimate for target $k = 19$ results from failed resolution in DOA-Doppler-delay.

Fig. 13: Simulated Environment 2: Fading coefficient estimates (magnitude and phase).

Having estimated all other target model parameters, path fading coefficients can then be estimated directly. Computer simulations have been presented to demonstrate the validity and effectiveness of these methods.

**APPENDIX A**

**PROOF OF EQUATION 13**

In this appendix, the following relationships will be used:

\[
\mathbf{a}^H \mathbf{b} = (a^* \odot b)^T \frac{1}{2} \quad (54)
\]

\[
\operatorname{vec}(\mathbf{A} \mathbf{B} \mathbf{C}) = (\mathbf{C}^T \otimes \mathbf{A}) \operatorname{vec} (\mathbf{B}) \quad (55)
\]

\[
(a \otimes b) \odot (c \otimes d) = (a \otimes c) \odot (b \otimes d) \quad (56)
\]

Thus, we proceed by rearranging Equation 5 as follows (where a number above an equals sign denotes the relevant relationship):

\[
\mathbf{x}(t) = \sum_{k=1}^{K} \beta_k \exp(j2\pi \mathbf{F}_k t) \frac{1}{2} \left( \mathbf{S}_k^* \otimes \mathbf{a}[n] \right)^T \mathbf{c}(t-\tau_k) + \mathbf{n}(t)
\]

\[
\sum_{k=1}^{K} \frac{1}{2} \beta_k \exp(j2\pi \mathbf{F}_k t) \left( \mathbf{S}_k^* \otimes \mathbf{a}[n] \right)^T \mathbf{c}(t-\tau_k) + \mathbf{n}(t)
\]

which leads directly to Equation 13.

**APPENDIX B**

**GEOMETRIC PROPERTIES OF VIRTUAL ARRAY MANIFOLD**

Since the virtual array manifold lies on a hypersphere of radius $\|\mathbf{S}_v\| = \sqrt{NN}$ (compared to $\|\mathbf{S}\| = \sqrt{N}$), we intuitively expect that the virtual manifold should be able to
accommodate uncertainty hyperspheres whose radii are at least $\sqrt{N}$ larger. Indeed, this will now be proven by studying the circular approximation of the virtual array manifold.

For the sake of this discussion, it is convenient to define the \((N \times 1)\) real vector \(\mathbf{A}(p) \triangleq -[\mathbf{r}_x, \mathbf{r}_y, \mathbf{c}_x, \mathbf{c}_y]^T \mathbf{\kappa}(p)\), such that:

\[
\mathbf{S}(p) = \exp(\mathbf{j} \mathbf{A}(p)) \tag{57}
\]

Furthermore, a dot over a symbol will be used to denote differentiation with respect to \(p\).

Following the methods outlined in [12, Equation 30] for evaluating the properties of extended array manifolds, it can be shown from Equation 20 that:

\[
\mathbf{\kappa}_v(p) = \sqrt{N} \mathbf{\kappa}_v^2(p) + N \mathbf{\kappa}_v^2(p) \tag{58}
\]

where \(\mathbf{\kappa}_v(p)\) is the rate of change of arc length associated with the transmit array’s manifold.

It can be shown that the curvature of the circular approximation of the virtual manifold (which takes into account the inclination angle, \(\zeta_v\)) is given by:

\[
\mathbf{\kappa}_{1,v} \triangleq \mathbf{\kappa}_{1,v} \sin \zeta_v
\]

\[
= \frac{1}{\mathbf{\kappa}_v^2} \left[ \left( \left\| \mathbf{\dot{A}}_v \right\|^2 + \left\| \mathbf{\ddot{A}}_v \right\|^2 - \mathbf{\kappa}_v^2 \right) - \left( \frac{\mathbf{\kappa}_v^2 \mathbf{A}_v^3}{\mathbf{\kappa}_v^4} \right)^2 \right] \tag{59}
\]

Since the virtual manifold lies on a hypersphere of significantly larger radius, its principal curvature tends to be smaller than in the receiver-only case. Therefore, it is important to prove that overall resolution/estimation performance is dominated by the improvement offered by \(\mathbf{\kappa}_v(p)\). After a series of rearrangements, it can be shown that:

\[
\frac{\mathbf{\kappa}_v^2}{\mathbf{\kappa}_v^2} \left( \frac{\mathbf{\dot{A}}_v^4}{N} - \frac{1}{N} \right) = \frac{\mathbf{\kappa}_v^2}{\mathbf{\kappa}_v^2} \left( \frac{\mathbf{\kappa}_v^2}{N} - \frac{1}{N} \right) + N \mathbf{\kappa}_v^4 \left( \frac{\mathbf{\kappa}_v^2}{N} - \frac{1}{N} \right) + 4 \mathbf{\kappa}_v^2 \mathbf{\dot{A}}_v^2
\]

\[
+ \frac{\mathbf{\kappa}_v^4}{N} \left( \frac{\mathbf{\kappa}_v^2}{N} - \frac{1}{N} \right)^2 \left( \frac{\mathbf{\kappa}_v^2}{N} - \frac{1}{N} \right) \left( \frac{\mathbf{\kappa}_v^2}{N} - \frac{1}{N} \right) \left( \frac{\mathbf{\kappa}_v^2}{N} - \frac{1}{N} \right) \right]
\]

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Harry Commin received the M.Eng. degree in electrical and electronic engineering from Imperial College London in 2009. He is currently pursuing the Ph.D. degree with the Communications and Signal Processing Group, Department of Electrical and Electronic Engineering, Imperial College London. His primary research interests are in the area of array signal processing, particularly superresolution parameter estimation, space-time array processing, performance bounds, differential geometry in array processing and arrayed MIMO radar.

Athanassios Manikas (SM’02) holds the Chair of Communications and Array Processing in the Department of Electrical and Electronic Engineering, Imperial College London and is currently the Technical Chair/Lead of the University Defence Research Centre in Signal Processing (DSTL/EPSRC). He is leading a strong group of researchers at Imperial College and has successfully supervised more than 30 Ph.D.’s and more than 100 Master’s project-students. He is on the Editorial Board of the IET Proceedings in Signal Processing and has held a number of research consultancies for the European Union, industry and government organizations. Also, he has had various technical chairs at international conferences and has been a TPC member of major IEEE conferences. He has served as an Expert Witness in the High Court of Justice (U.K.) and as a member of the Royal Society’s International Fellowship Committee (January 1, 2008 until December 31, 2010). He is a Fellow of IET (U.K) and a Chartered Engineer.