Superresolution Multitarget Parameter Estimation in MIMO Radar

Kai Luo, Student Member, IEEE, and Athanassios Manikas, Senior Member, IEEE

Abstract—This paper is concerned with a multiple-input—multiple-output (MIMO) radar operating in an environment with two or more closely located targets. In this scenario, mutual target interference is a serious problem for multitarget parameter estimation, reducing the performance of existing methods such as least squares, Capon, and amplitude and phase estimation. In contrast to previous methods where the overall effect of mutual target interference is treated as “noise,” in this paper, two methods for suppressing this interference are proposed. The first is based on a constrained optimization problem that provides an iterative method. The second involves a novel nonlinear optimization based on a cost function of targets’ directions which is solved using the biogeography-based-optimization algorithm. The performances of both the proposed approaches are evaluated via computer simulation studies and shown to outperform existing methods.

Index Terms—Beamforming, interference suppression, MIMO radar, multitarget, parameter estimation.

NOTATIONS

\( a, A \) Scalar.

\( a, A \) Column Vector.

\( A \) Matrix.

\( 0_N \) \( N \times 1 \) vector of zeros.

\( I_N \) \( N \times N \) identity matrix.

\( (\cdot)^T, (\cdot)^H \) Transpose and Hermitian transpose.

\( (\cdot)^* \) Complex conjugate.

\( |a| \) Absolute value of \( a \).

\( \|a\| \) Elementwise absolute value of \( a \).

\( \|a\| \) Euclidean norm of \( a \).

\( \odot, \otimes \) Hadamard and Kronecker product.

\( \mathcal{E}\{\cdot\} \) Expectation operator.

\( \text{tr}\{\cdot\} \) Trace of \( A \).

\( \text{Re}\{\cdot\} \) Real part of a complex number.

\( \text{diag}\{a\} \) Diagonal matrix whose diagonal elements are the elements of \( a \).

\( \text{diag}\{A\} \) Column vector consisting of the diagonal elements of \( A \).

\( \exp\{a\} \) Elementwise exponential of vector \( a \).

\( \mathbb{R}, \mathbb{C} \) Field of real and complex numbers.

\( \text{vec}\{A\} \) Vectorization of \( A \).

I. INTRODUCTION

RECENTLY, significant attention has been drawn to MIMO radars [1]–[3]. The concept of MIMO radar in recent articles refers to a radar system emitting independent (usually orthogonal) waveforms through the transmit (Tx) array and processing echoes received by the receive (Rx) array. According to the array configuration, MIMO radars can mainly be classified as those deploying large aperture arrays [3] or small aperture arrays [2] at both the Tx and Rx. The first type deals with an extended target model which contains rich scatterers. The large aperture arrays at both the Tx and Rx enable the MIMO radar to view the different aspects of the target from different angles simultaneously. In other words, the large aperture arrays exploit the spatial diversity of the target’s radar cross section (RCS) to reduce the scintillation effects. Rather than distributing Tx and Rx antennas in a large area, the other type employs small aperture antenna arrays at both the Tx and Rx so that the MIMO radar views the same aspect of a target from the same angle (monostatic radar) or different angles for Tx and Rx arrays ( bistatic radar). In this case, point targets in the far field of a MIMO radar are assumed. With the transmit waveform diversity, MIMO radar with small aperture arrays at both the Tx and Rx provides flexible transmit beam pattern design [4], [5], achieves better robustness performance [6], and enhances the ability of multitarget localization [7]. This paper is concerned with monostatic MIMO radar, i.e., colocated MIMO radar in which the direction of departure and the direction of arrival (DOA) for the target are the same.

Target localization is one of the most important issues in radar sensing applications such as localizing aircraft in airspace by air defense radar systems, localizing the hostages and terrorists inside rooms or buildings by through-the-wall radar systems [8], or localizing land mines which are on or below the surface by ground-penetrating-radar systems [9]. The proposed approach in this paper is not based on mapping the electromagnetic received array signals to a 2D or 3D image, which is then used for target localization as in [10] and [11]. Instead, the target localization is carried out by directly exploiting the spatiotemporal properties of the received electromagnetic array signals without the need of image formation and its associated problems. Furthermore, although it is based on “narrow-band” and “far-field” (i.e., plane-wave) propagation models, where the Tx and Rx manifold vectors are given by (5) and (6), the proposed approach has the potential to be extended to wideband and/or near-field radar sensing (e.g., [12]–[14]) for multitarget

Manuscript received April 23, 2012; revised August 31, 2012; accepted October 14, 2012.

The authors are with the Department of Electrical and Electronic Engineering, Imperial College London, SW7 2AZ, London, U.K. (e-mail: kai.luo07@imperial.ac.uk; a.mankas@imperial.ac.uk).

Color versions of one or more of the figures in this paper are available online at http://ieeexplore.ieee.org.

Digital Object Identifier 10.1109/TGRS.2012.2226466

0196-2892/12$31.00 © 2012 IEEE
localization, using spherical wave propagation manifold vectors.

In colocated MIMO radar, the DOA and complex path gain are two crucial parameters to be estimated for multiple target localization. The DOA determines the direction of the target with respect to the radar, and the complex path gain is related to the RCS of the target as well as the path loss, the antenna gain, and the effect of the Doppler shift for the moving target. This paper is focused on the estimation of these two parameters. Many DOA and path gain estimation algorithms for passive radars can be applied directly in MIMO radar [7], [15]–[20]. In [15], least squares (LS), Capon, and amplitude and phase estimation (APES) are applied in MIMO radar systems to jointly estimate DOAs and complex path gains while generalized likelihood ratio test (GLRT) is applied for the DOA estimation of multiple targets, particularly in hostile environments. According to the comparison in [15], Capon, APES, and GLRT outperform the LS estimator. Capon provides accurate DOA estimates, and APES gives accurate complex path gain estimates, while GLRT has good anti-jamming capabilities. To obtain the benefits of both Capon and APES, in [16], the Capon-and-APES (CAPES) method estimates the DOAs of targets by Capon’s method and then provides estimates of the path gains of these DOAs via the APES method. In [17], an alternative approach, named Capon-and-AML (CAML), which utilizes Capon for the DOA estimation and an approximate maximum likelihood (AML) method for the path gain estimation, is presented.

However, as shown in [17], the performance of all these methods decreases significantly when targets are close together in space. The APES method may fail to resolve these targets. While the LS, Capon, CAPES, and CAML may resolve all the targets, the accuracy of the DOA estimation will degrade severely, which will, in turn, affect the accuracy of the estimates of complex path gains. This effect occurs because the small angular separations among multiple targets lead to high mutual target interference (i.e., mutual interferences among the echoes from the targets), which is not accounted for in these methods. In this paper, multitarget parameter estimation in the presence of these high interferences is addressed. Inspired by the APES method, an optimization problem is formulated which involves the suppression of these mutual interferences for multitarget parameter estimation. Two methods are proposed to solve this problem. The first is a 1-D iterative method which uses the suboptimal solutions of these constrained optimization problems with respect to the complex path gains of the targets and the Rx beamforming matrix separately. The second method utilizes the optimal solution of the complex path gains (connecting the complex path gains to DOAs) to form a nonlinear cost function of DOAs of targets which gives the estimates by employing the biogeography-based optimization (BBO) [21] algorithm. By suppressing the mutual interferences in the multitarget parameter estimation problem, both the proposed methods outperform the existing ones. Numerical results validate the performance improvement achieved by these proposed methods.

The remainder of this paper is organized as follows. Section II provides the MIMO radar system model. In Sections III and IV, the mutual target interference suppressed optimization problem for multitarget parameter estimation is developed, and two estimators are proposed. Following this, the numerical results are presented in Section V. Finally, the conclusions are drawn in Section VI.

II. PROBLEM FORMULATION

Consider a narrow-band MIMO radar system with an array of \( N_{Tx} \) antennas at the transmitter and an array of \( N_{Rx} \) antennas at the receiver. Assume that both the Tx and Rx arrays have a common reference point and are near each other in space (i.e., monostatic radar) such that each target can be considered to be at the same location in the far field with respect to both the Tx and Rx arrays. Without loss of generality, assume that the Tx array, the Rx array, and the targets are located in the same 2-D space.

A set of \( N_{Tx} \) modulated waveforms \( m(t) \in \mathbb{C}^{N_{Tx} \times 1} \) described by

\[
    m(t) = \sum_{n=1}^{L} q(n) \epsilon(t - (n - 1)2N_c T_c)
\]

is emitted through the array elements in the Tx array. Here, \( \{ q(n) \}_{n=1}^{L} \) denotes \( N_{Tx} \) linear independent sequences which are spread by the spreading waveform \( \epsilon(t) \)

\[
    \epsilon(t) = \sum_{i=-1}^{N_{Rc}} \alpha[i] p(t - (i - 1) T_c)
\]

where \( p(t) \) represents the chip-shaping waveform of duration \( T_c \) and \( \{ \alpha[i] \}, i = 1, \ldots, N_{Rc} \) is a zero-padded \( \mathbb{Z} \)-sequence denoted by \( \alpha \), i.e.,

\[
    \alpha \triangleq [\alpha[1], \alpha[2], \ldots, \alpha[N_{Rc}], 0]^{T}.
\]

Assume that there are \( K \) targets which are located at distinct directions \( \theta_k, k = 1, 2, \ldots, K \), with respect to the radar’s common reference point and different relative delays \( \tau_k, k = 1, 2, \ldots, K \), with respect to the reference clock at the Rx. Consider that the transmitted waveforms \( m(t) \) from the Tx array are reflected by the multiple targets and received by the Rx array. The propagation of the signal vector transmitting from the Tx array to each of the targets and reflecting to the Rx array is shown in Fig. 1.

The equivalent baseband received signal (see Point (B) in Fig. 1) can be written as

\[
    \tilde{v}(t) = \sum_{k=1}^{K} \beta_k S_{R_k}(\theta_k) S_{T_k}(\theta_k) m(t - \tau_k) + \eta(t)
\]

where \( S_{R_k}(\theta_k) \in \mathbb{C}^{N_{Rx} \times 1} \) and \( S_{T_k}(\theta_k) \in \mathbb{C}^{N_{Rx} \times 1} \) denote the manifold vectors of the Tx and Rx arrays, respectively, for the \( k \)-th target, which are defined as

\[
    S_{R_k}(\theta_k) \triangleq \exp \left(-j[\tau_x, \tau_y, \tau_z]_{R_k}(k|\theta_k) \right)
\]

\[
    S_{T_k}(\theta_k) \triangleq \exp \left(j[\tau_x, \tau_y, \tau_z]_{T_k}(k|\theta_k) \right)
\]

where \( \theta_k \) is the direction-of-departure or DOA of the \( k \)-th target. Furthermore, in (5) and (6), the matrices \( [\tau_x, \tau_y, \tau_z]_{Rx} \) and \( [\tau_x, \tau_y, \tau_z]_{Tx} \) denote the Cartesian coordinates of the Rx and Tx antenna-array elements, respectively, and

\[
    k(\theta_k) = \frac{2\pi F_c}{c} \left[ \cos \theta_k, \sin \theta_k, 0 \right]^{T}
\]
is the wavenumber vector (pointing toward the target at direction \( \theta_k \)) with \( F_c \) denoting the carrier frequency and \( c \) denoting the velocity of light. Furthermore, in (4), the parameter \( \beta_k \) denotes the complex path gain associated with the \( k \)-th target, and \( \tilde{u}(t) \) represents the circularly symmetric complex Gaussian noise with \( \tilde{u}(t) \sim N(0, \sigma^2_{\tilde{N}} I_{N_c}) \).

The discretized received signal \( \mathbf{X} \in \mathbb{C}^{N_{Rx} \times 2N_c L} \) according to (4) is given by

\[
\mathbf{X} = \sum_{k=1}^{K} \beta_k \mathbf{S}_{Rx}(\theta_k) \mathbf{S}_{Tx}^H(\theta_k) \left( \mathbf{M} \otimes \left( \mathbf{J}^k \mathbf{\xi} \right)^T \right) + \tilde{\mathbf{N}} \tag{8}
\]

where \( \mathbf{M} \in \mathbb{C}^{N_{Rx} \times \mathbb{R}} \) represents

\[
\mathbf{M} \triangleq [a[1], a[2], \ldots, a[L]]. \tag{9}
\]

Furthermore, \( \mathbf{J} \in \mathbb{R}^{2N_c \times 2N_c} \) is the downshift matrix, and \( l_k \) is the quantized relative delay path of the \( k \)-th target, which are defined as

\[
\mathbf{J} \triangleq \begin{bmatrix}
0 & 0 \\
I_{2N_c-1} & 0
\end{bmatrix} \tag{10}
\]

and

\[
l_k \triangleq \begin{bmatrix}
\tau_k \\
\frac{c l_k}{T_c}
\end{bmatrix}, \quad k = 1, 2, \ldots, K. \tag{11}
\]

Thus, when \( \mathbf{J}^k \) operates on a column vector, it downshifts the elements of the vector by \( l_k \) elements, which is an effective way to model the relative delays of targets’ echoes. In order to exploit the relative delays, a space time array (STAR) architecture at the Rx can be employed by using a bank of tapped-delay lines of length \( 2N_c \) (see [22]). This is equivalent to partitioning the received signal matrix \( \mathbf{X} \in \mathbb{C}^{N_{Rx} \times 2N_c L} \) into \( L \) block submatrices as follows:

\[
\mathbf{X} = [\tilde{\mathbf{X}}_1, \tilde{\mathbf{X}}_2, \ldots, \tilde{\mathbf{X}}_L] \tag{12}
\]

and then vectorizing each submatrix \( \tilde{\mathbf{X}}_k \) (row vectorization) to form a new received signal matrix \( \tilde{\mathbf{X}} \in \mathbb{C}^{2N_{Rx} N_c \times L} \) as

\[
\tilde{\mathbf{X}} = \left\{ \text{vec} \{ \tilde{\mathbf{X}}_1 \}, \text{vec} \{ \tilde{\mathbf{X}}_2 \}, \ldots, \text{vec} \{ \tilde{\mathbf{X}}_L \} \right\}. \tag{13}
\]

Equation (13) can be expressed as a function of the STAR manifold vector

\[
\tilde{\mathbf{S}}_{Rx}(\theta_k, l_k) \triangleq \mathbf{S}_{Rx}(\theta_k) \otimes (\mathbf{J}^k \mathbf{\xi}) \in \mathbb{C}^{2N_{Rx} N_c \times 1} \tag{14}
\]

i.e.,

\[
\tilde{\mathbf{X}} = \sum_{k=1}^{K} \beta_k \tilde{\mathbf{S}}_{Rx}(\theta_k, l_k) \mathbf{S}_{Tx}^H(\theta_k) \mathbf{M} + \tilde{\mathbf{N}} \tag{15}
\]

where \( \tilde{\mathbf{N}} \in \mathbb{C}^{2N_{Rx} N_c \times L} \) is the corresponding noise matrix with the same statistical properties as \( \tilde{\mathbf{N}} \in \mathbb{C}^{N_{Rx} \times 2N_c L} \).

It is important to point out that: existing methods such as LS, Capon, and APES are based on the following signal model [16]:

\[
\mathbf{\tilde{X}} = \beta_d \tilde{\mathbf{S}}_{Rx}(\theta_d, l_d) \mathbf{S}_{Tx}^H(\theta_d) \mathbf{M} + \mathbf{\tilde{Z}} \tag{16}
\]

where \( \mathbf{\tilde{Z}} \) represents the residual term, which includes the echoes from the other targets and noise, i.e.,

\[
\mathbf{\tilde{Z}} = \sum_{k=1}^{K} \beta_k \tilde{\mathbf{S}}_{Rx}(\theta_k, l_k) \mathbf{S}_{Tx}^H(\theta_k) \mathbf{M} + \tilde{\mathbf{N}}. \tag{17}
\]

This implies that the effect of the mutual interferences among echoes from multiple targets is considered as additive white Gaussian noise which is not appropriate, particularly when the targets are close to each other. In this case, the performance of these methods degrades significantly, particularly when the number of snapshots is small.

It is worth noting that the reconstructed received signal \( \tilde{\mathbf{X}} \) can be rewritten in a more compact form according to (15)

\[
\tilde{\mathbf{X}} = \tilde{\mathbf{S}}_{Rx}(\theta, l) \text{diag} \{ \beta \} \mathbf{S}_{Tx}^H(\theta) \mathbf{M} + \tilde{\mathbf{N}} \tag{18}
\]

where

\[
\tilde{\mathbf{S}}_{Rx}(\theta, l) = [\tilde{\mathbf{S}}_{Rx}(\theta_1, l_1), \tilde{\mathbf{S}}_{Rx}(\theta_2, l_2), \ldots, \tilde{\mathbf{S}}_{Rx}(\theta_K, l_K)] \tag{19}
\]

\[
\mathbf{S}_{Tx}(\theta) = [\mathbf{S}_{Tx}(\theta_1), \mathbf{S}_{Tx}(\theta_2), \ldots, \mathbf{S}_{Tx}(\theta_K)] \tag{20}
\]

\[
\theta = [\theta_1, \theta_2, \ldots, \theta_K]^T \tag{21}
\]

\[
\beta = [\beta_1, \beta_2, \ldots, \beta_K]^T \tag{22}
\]

\[
l = [l_1, l_2, \ldots, l_K]^T. \tag{23}
\]

For simplicity, the relative delays \( l \) of the targets are assumed to be pre estimated. Thus, \( \theta \) and \( \beta \) are the two unknown parameter vectors to be estimated in such a MIMO radar system. The parameter vector \( \theta \) gives the directions of the multiple targets, while the parameter vector \( \beta \) is related to the ranges and the cross section of these targets as well as the Doppler shifts for moving targets. Moreover, for notation convenience, \( \tilde{\mathbf{X}} \) and \( \tilde{\mathbf{S}}_{Rx}(\theta) \) will be redefined as \( \mathbf{\tilde{X}} \) and \( \mathbf{S}_{Rx}(\theta) \), respectively, for the remainder of this paper. Thus, the reconstructed received signal \( \mathbf{\tilde{X}} \) in (18) is expressed as

\[
\mathbf{\tilde{X}} = \mathbf{S}_{Rx}(\theta) \text{diag} \{ \beta \} \mathbf{S}_{Tx}^H(\theta) \mathbf{M} + \mathbf{\tilde{N}}. \tag{24}
\]

It is worth noting that, when transmit waveforms are without spreading, (24) can represent the signal model in which the relative delays of all the targets are negligible.
III. FRAMEWORK OF INTERFERENCE
SUPPRESSED OPTIMIZATION

Based on the signal model of (24), a constrained optimization problem, which suppresses mutual multtarget interferences, is formulated for the estimation of \( \beta \) and \( \beta_0 \). First, let us define the mean square error (mse) matrix \( \mathbf{E} \) of the echoes from multiple targets at Point (B) in Fig. 1 as

\[
\mathbf{E} \triangleq \frac{1}{L} \left[ \mathbf{W}_{Rx}^H \mathbf{X} - \text{diag} \{ \beta \} \mathbf{S}_{Tx}^H(\theta) \mathbf{M} \right]^H \cdot \left[ \mathbf{W}_{Rx}^H \mathbf{X} - \text{diag} \{ \beta \} \mathbf{S}_{Tx}^H(\theta) \mathbf{M} \right] 
\]  

(25)

where the matrix \( \mathbf{W}_{Rx} \in \mathbb{C}^{N_{Rx} \times N_s \times K} \) is the Rx beamforming matrix, of which the goal is to suppress both the mutual interferences and noise while keeping the echoes undistorted. This can be formulated using the following constrained optimization problem:

\[
\begin{align*}
& \min_{\beta, \mathbf{W}_{Rx}} \quad \text{Tr}\{\mathbf{E}\} & \quad (26a) \\
& \text{subject to} \quad \mathbf{S}_{Tx}^H(\theta) \mathbf{W}_{Rx} = \mathbf{I}_K. & \quad (26b)
\end{align*}
\]

This constrained multivariable optimization problem can be solved by dividing it into two separate optimization problems which are with respect to the two variables \( \beta \) and \( \mathbf{W}_{Rx} \), respectively, and then combining the solutions to these two separate optimization problems.

A. Optimal Solution of Complex Path Gain \( \beta \)

First, consider the optimization problem with respect to the path gain vector \( \beta \) with \( \mathbf{W}_{Rx} \) fixed, i.e.,

\[
\min_{\beta} \quad \text{Tr}\{\mathbf{E}\}. 
\]

In this case, the cost function \( \xi_{\beta} \triangleq \text{Tr}\{\mathbf{E}\} \) can be written as

\[
\begin{align*}
\xi_{\beta} = & - \text{tr} \left\{ \mathbf{W}_{Rx}^H \mathbf{R}_{xx} \mathbf{W}_{Rx} \right\} - \text{tr} \left\{ \mathbf{S}_{Tx}^H(\theta) \mathbf{R}_{mm}^H \mathbf{W}_{Rx} \text{diag} \{ \beta \} \right\} \\
& + \text{tr} \left\{ \mathbf{S}_{Tx}^H(\theta) \mathbf{R}_{mm} \mathbf{S}_{Tx}(\theta) \text{diag} \{ \beta \} \text{diag} \{ \beta \} \right\} 
\end{align*}
\]

(28)

where

\[
\begin{align*}
\mathbf{R}_{mm} & \triangleq \frac{1}{L} \mathbf{MM}^H \in \mathbb{C}^{N_{Tx} \times N_{Tx}} \quad (29) \\
\mathbf{R}_{xx} & \triangleq \frac{1}{L} \mathbf{XX}^H \in \mathbb{C}^{N_{Rx} \times N_{Rx} \times N_{Rx}} \quad (30) \\
\mathbf{R}_{x} & \triangleq \frac{1}{L} \mathbf{XX} \in \mathbb{C}^{N_{Rx} \times N_{Rx} \times N_{Rx} \times N_{Rx}} 
\end{align*}
\]

To solve the above optimization problem, some properties of the operator \( \text{diag}\{\cdot\} \) are given in the following two lemmas, the proof of which is in Appendix A:

Lemma 1: \( \text{diag} \{ A \text{diag} \{ b \} \} C = (A \otimes C^T) b \)  

Lemma 2: \( \frac{\partial }{\partial \beta} \text{tr} \{ \mathbf{D} \text{diag} \{ \beta \} \} = \text{diag} \{ \mathbf{D} \} \)  

(32)

(33)

where \( A \in \mathbb{C}^{M \times N}, \quad C \in \mathbb{C}^{N \times M}, \quad \mathbf{D} \in \mathbb{C}^{N \times N}, \) and \( b \in \mathbb{C}^{N \times 1} \).

Setting the derivative of \( \xi_{\beta} \) with respect to \( \beta \) to zero yields

\[
\begin{align*}
\frac{\partial \xi_{\beta} }{\partial \beta} & = \left( \mathbf{S}_{Tx}^H(\theta) \mathbf{R}_{mm} \mathbf{S}_{Tx}(\theta) \odot \mathbf{I}_K \right) \mathbf{S}_{Tx}^H(\theta) \mathbf{R}_{mm}^H \mathbf{W}_{Rx} \\
& \quad - \text{diag} \left\{ \mathbf{S}_{Tx}^H(\theta) \mathbf{R}_{XX} \mathbf{W}_{Rx} \right\}
\end{align*}
\]

(34)

Therefore, the solution for the optimization problem in (27) is given as

\[
\beta = \frac{1}{N_{Tx}} \text{diag} \left\{ \mathbf{W}_{Rx}^H \mathbf{R}_{xx} \mathbf{S}_{Tx}(\theta) \right\}. 
\]

(35)

Note that (35) can also be expressed in terms of individual targets as

\[
\beta_k(\theta_k) = \frac{1}{N_{Tx}} \mathbf{w}_{Rx,k}^H \mathbf{R}_{xx} \mathbf{S}_{Tx}(\theta_k), \quad k = 1, \ldots, K
\]

(36)

where

\[
\mathbf{w}_{Rx} = [\mathbf{w}_{Rx,1}, \mathbf{w}_{Rx,2}, \ldots, \mathbf{w}_{Rx,K}]. 
\]

B. Optimal Rx Beamforming Matrix \( \mathbf{W}_{Rx} \)

With fixed \( \beta \), the constrained optimization problem described by (26) can be rewritten as

\[
\begin{align*}
& \min_{\mathbf{W}_{Rx}} \quad \text{Tr}\{\mathbf{E}\} & \quad (38a) \\
& \text{subject to} \quad \mathbf{S}_{Tx}^H(\theta) \mathbf{W}_{Rx} = \mathbf{I}_K. & \quad (38b)
\end{align*}
\]

Using the following two vectorization identities [23]:

\[
\begin{align*}
\text{tr} \{ \mathbf{A}^H \mathbf{B} \} & = \text{vec} \{ \mathbf{A}^H \} \text{vec} \{ \mathbf{B} \} \\
\text{vec} \{ \mathbf{A}^H \} & = (\mathbf{I}_N \otimes \mathbf{A})^H \text{vec} \{ \mathbf{B} \}
\end{align*}
\]

(39)

(40)

where \( \mathbf{A}, \mathbf{B} \in \mathbb{C}^{M \times N} \). The constraint given by (38b) can be equivalently rewritten as

\[
(\mathbf{I}_K \otimes \mathbf{S}_{Tx}(\theta))^H \text{vec} \{ \mathbf{W}_{Rx} \} = \text{vec} \{ \mathbf{I}_K \}
\]

(41)

and the objective function \( \xi_{\mathbf{W}_{Rx}} \triangleq \text{tr}\{\mathbf{E}\} \) can be written as

\[
\begin{align*}
\xi_{\mathbf{w}_{Rx}} = & \text{vec} \{ \mathbf{W}_{Rx} \}^H (\mathbf{I}_K \otimes \mathbf{R}_{xx}) \text{vec} \{ \mathbf{W}_{Rx} \} \\
& - \text{vec} \{ \mathbf{R}_{xx} \mathbf{S}_{Tx}(\theta) \text{diag} \{ \beta \} \}^H \text{vec} \{ \mathbf{W}_{Rx} \} \\
& - \text{vec} \{ \mathbf{W}_{Rx} \}^H \text{vec} \{ \mathbf{R}_{xx} \mathbf{S}_{Tx}(\theta) \text{diag} \{ \beta \} \} \\
& + \text{tr} \{ \text{diag} \{ \beta \} \mathbf{S}_{Tx}^H(\theta) \mathbf{R}_{mm} \mathbf{S}_{Tx}(\theta) \text{diag} \{ \beta \} \}.
\end{align*}
\]

(42)

For convenience, let us define the following terms:

\[
\begin{align*}
\mathbf{G}_{Rx} & \triangleq \text{vec} \{ \mathbf{W}_{Rx} \} \in \mathbb{C}^{2KN_{Rx} N_c \times 1} \\
\mathbf{S}_{Rx}(\theta) & \triangleq \mathbf{I}_K \otimes \mathbf{S}_{Rx}(\theta) \in \mathbb{C}^{2KN_{Rx} N_c \times K K} \\
\mathbf{R}_{xx} & \triangleq \mathbf{I}_K \otimes \mathbf{R}_{xx} \in \mathbb{C}^{2KN_{Rx} N_c \times 2KN_{Rx} N_c} \\
\mathbf{v} & \triangleq \text{vec} \{ \mathbf{I}_K \} \in \mathbb{C}^{KK \times 1}
\end{align*}
\]

(43)

(44)

(45)

(46)
\[
\begin{align*}
\mathbf{u} & \doteq \text{vec} \left\{ \mathbb{E}_{\mathbf{x}} \mathbf{S}_{\mathbf{x}}(\theta) \text{diag} \left( \mathbf{\beta}^* \right) \right\} \in \mathbb{C}^{2KN \times 1} \\
\mathbf{C} & \doteq \text{tr} \left\{ \text{diag} \left( \mathbf{\beta} \right) \mathbf{S}_{\mathbf{x}}^H(\theta) \mathbb{E}_{\mathbf{x}} \mathbf{S}_{\mathbf{x}}(\theta) \text{diag} \left( \mathbf{\beta}^* \right) \right\}.
\end{align*}
\]

Using (33)-(38), the constrained optimization problem in (38) can be expressed as
\[
\begin{align}
\min_{\mathbf{w}_{\text{Rx}}} & \quad \tilde{\varepsilon}_{\text{w}_{\text{Rx}}} \\
\text{subject to} & \quad \tilde{\mathbf{w}}_{\text{Rx}}^H(\theta) \mathbf{w}_{\text{Rx}} = \mathbf{u}
\end{align}
\]

where
\[
\tilde{\varepsilon}_{\text{w}_{\text{Rx}}} = \tilde{\mathbf{w}}_{\text{Rx}}^H(\theta) \mathbf{w}_{\text{Rx}} - \mathbf{u}^H \mathbf{w}_{\text{Rx}} - \tilde{\mathbf{w}}_{\text{Rx}}^H(\theta) \mathbf{u} + C.
\]

Equation (49) is a typical equality constrained quadratic optimization problem. It can be transformed to an unconstrained optimization problem based on the cost function \( \tilde{\varepsilon}_{\text{w}_{\text{Rx}}} \).
\[
\tilde{\mathbf{e}}_{\text{w}_{\text{Rx}}} = \tilde{\varepsilon}_{\text{w}_{\text{Rx}}} + \text{Re} \left\{ \Delta^H \left( \mathbf{S}_{\mathbf{x}}^H(\theta) \mathbf{w}_{\text{Rx}} - \mathbf{u} \right) \right\}
\]

where \( \Delta \in \mathbb{C}^{2KN \times 1} \) is the vector of Lagrange multipliers. If the derivative of this cost function with respect to \( \mathbf{w}_{\text{Rx}} \) is set to zero, we have
\[
\frac{\partial \tilde{\mathbf{e}}_{\text{w}_{\text{Rx}}}^T}{\partial \mathbf{w}_{\text{Rx}}} = \begin{bmatrix} \tilde{\mathbf{w}}_{\text{Rx}}^H(\theta) \tilde{\mathbf{w}}_{\text{Rx}} - \mathbf{u}^T + \frac{1}{2} \Delta^H \mathbf{S}_{\mathbf{x}}^H(\theta) \end{bmatrix}^T = 0,
\]

which provides the vectorized weight matrix \( \tilde{\mathbf{w}}_{\text{Rx}} \) as a function of the vector of the Lagrange multipliers \( \Delta \), i.e.,
\[
\tilde{\mathbf{w}}_{\text{Rx}} = \mathbf{w}_{\text{Rx}} \left( \mathbf{w}_{\text{Rx}} - \frac{1}{2} \mathbf{S}_{\mathbf{x}}(\theta) \Delta \right).
\]

Substituting (51) into (49) yields the solution for the vector of Lagrange multipliers \( \Delta \) as follows:
\[
\Delta = \left( \mathbf{S}_{\mathbf{x}}^H(\theta) \mathbf{w}_{\text{Rx}}^{-1} \mathbf{S}_{\mathbf{x}}(\theta) \right)^{-1} \left( \mathbf{S}_{\mathbf{x}}^H(\theta) \mathbf{w}_{\text{Rx}}^{-1} \tilde{\mathbf{w}}_{\text{Rx}} - \mathbf{u} \right).
\]

Inserting (54) into (53) provides the solution \( \tilde{\mathbf{w}}_{\text{Rx}} \), which is given by
\[
\begin{align*}
\tilde{\mathbf{w}}_{\text{Rx}} = & \begin{bmatrix} \mathbf{w}_{\text{Rx}} - \tilde{\mathbf{w}}_{\text{Rx}}(\theta) \left( \mathbf{S}_{\mathbf{x}}^H(\theta) \tilde{\mathbf{w}}_{\text{Rx}}^{-1} \mathbf{S}_{\mathbf{x}}(\theta) \right)^{-1} \\
\end{bmatrix}^T \\
\times & \left( \mathbf{S}_{\mathbf{x}}^H(\theta) \tilde{\mathbf{w}}_{\text{Rx}}^{-1} \tilde{\mathbf{w}}_{\text{Rx}} - \mathbf{u} \right).
\end{align*}
\]

Then, utilizing the properties of the vectorization operation, the optimal receive beamforming matrix \( \mathbb{W}_{\text{Rx}} \) can be reconstructed from the vectorized beamforming vector \( \tilde{\mathbf{w}}_{\text{Rx}} \). The final result of the matrix \( \mathbb{W}_{\text{Rx}} \) is given by (56), and a detailed proof is provided in Appendix B.

\[
\begin{align}
\mathbb{W}_{\text{Rx}} = & \begin{bmatrix} \tilde{\mathbf{w}}_{\text{Rx}}(\theta) \\
\mathbf{w}_{\text{Rx}}(\theta) \\
\end{bmatrix}^T \\
\times & \left( \mathbf{S}_{\mathbf{x}}^H(\theta) \mathbf{w}_{\text{Rx}}^{-1} \mathbf{S}_{\mathbf{x}}(\theta) \right)^{-1} \\
\times & \mathbf{S}_{\mathbf{x}} \mathbf{S}_{\mathbf{x}}^H(\theta) (\mathbf{S}_{\mathbf{x}}^H(\theta) \mathbf{w}_{\text{Rx}}^{-1} \mathbf{S}_{\mathbf{x}}(\theta))^{-1}.
\end{align}
\]

IV. Interference Suppressed Multitarget Parameter Estimation

Based on the derivation of the proposed optimization problem in Section III, two methods are proposed for the multitarget parameter estimation. One is the 1-D iterative method, and the other is the multidimensional optimal method. It is important to point out that the proposed methods assume that the number of targets is known or has been estimated using, for instance, Akaike Information Criterion [24], minimum description length [25], Gerschgorin disks [26], etc.

A. One-Dimensional Iterative Method (Proposed Method 1)

As derived in Sections III-A and III-B, the path gain \( \mathbf{\beta} \) in (35) [or its scalar form in (36)] and the Rx beamforming matrix \( \mathbb{W}_{\text{Rx}} \) in (36) are functions of each other. Hence, an iterative method can be devised for estimating the parameters \( \theta_k \) and \( \beta_k \), \( k = 1, \ldots, K \), of multiple targets as follows.

1) The data \( \mathbf{X} \) are collected from the Rx and formed as (24), and then, \( \mathbb{E}_{\mathbf{x}} \) and \( \mathbf{R}_{\mathbf{x}} \) are calculated according to (30) and (31).

2) The DOAs \( \hat{\theta}^{[0]} \) and complex path gains \( \hat{\beta}^{[0]} \) of multiple targets are initialized. For instance, the initial \( \hat{\theta}^{[0]} \) can be given by any existing estimator such as LS, Capon, CAPES, etc.

3) The initial Rx beamforming matrix \( \mathbb{W}_{\text{Rx}}^{[0]} \) can be formed by
\[
\mathbb{W}_{\text{Rx}}^{[0]} = \mathbf{S}_{\text{Rx}} \mathbf{S}_{\text{Rx}}^H(\hat{\theta}^{[0]})
\]
or following (56).

4) The multiple targets’ DOAs \( \hat{\theta}^{[k]} \), \( k = 1, \ldots, K \), are estimated based on the spectrum search of the amplitude of the path gain according to (36), i.e.,
\[
\hat{\theta}^{[k]} = \arg \max_{\theta} \left( \mathbb{W}_{\text{Rx},k}(\theta)^H \mathbb{E}_{\mathbf{x}} \mathbf{S}_{\mathbf{x}}(\theta) \right), \quad k = 1, \ldots, K
\]

where, according to (37), \( \mathbb{W}_{\text{Rx},k}(\theta) \), \( k = 1, \ldots, K \), denotes the columns of \( \mathbb{W}_{\text{Rx}}^{[k]} \). Alternatively, the vector formed expression for the DOA estimation according to (35) is given as
\[
\hat{\theta}^{[k]} = \arg \max_{\theta} \left( \text{diag} \left\{ \mathbb{W}_{\text{Rx},k}^H(\theta) \mathbb{E}_{\mathbf{x}} \mathbf{S}_{\mathbf{x}}(\theta) \right\} \right).
\]

5) Estimates of the complex path gains \( \hat{\beta}^{[k]} \) are given by (36) or (35) with estimated DOAs \( \hat{\theta}^{[k]} \).

6) Calculate the sum of the mses of the received signals at Point (B) in Fig. 1, i.e.,
\[
\epsilon^{[k]} \doteq \frac{1}{L} \mathbb{E} \left( \mathbf{X} - \mathbf{S}_{\text{Rx}}^H(\hat{\theta}^{[k]}) \text{diag} \left\{ \beta^{[k]} \right\} \mathbf{S}_{\text{Rx}}^H \right)^2.
\]
If $e_i^{[i]} \geq e_i^{[i-1]}$, then the current estimates $(\tilde{\theta}_i^{[i]}, \tilde{\beta}_i^{[i]})$ are replaced by $(\hat{\theta}_i^{[i]}, \hat{\beta}_i^{[i-1]})$ obtained from the previous estimation.

7) Using the estimated parameters $(\hat{\theta}_i^{[i]}, \hat{\beta}_i^{[i-1]})$ from Steps 4) and 5), the Rx beamforming matrix $\mathbf{W}_\text{Rx}^{[i]}$ is calculated via (56).

8) Update the index $[i]: [i] \rightarrow [i - 1]$.

9) Repeat Steps 4)–8) until a stopping criterion is satisfied.

B. Multidimensional Optimal Method Using BBO (Proposed Method 2)

Based on the optimization with respect to $\mathbf{W}_\text{Rx}$ and $\beta$ separately, a novel expression of $\beta$ as only a function of $\hat{\theta}$ can be derived by substituting (56) into (35). That is,

$$\hat{\beta}(\hat{\theta}) = \left( N_{\text{Tx}} \mathbf{I}_K - \mathbf{I}_K \otimes (\mathbf{S}_{\text{Rx}}^H(\hat{\theta}) \mathbf{R}_{\text{ex}}^{-1} \mathbf{R}_{\text{zx}} \mathbf{S}_{\text{Tx}}(\hat{\theta})) \right)^{-1} \cdot \text{diag} \left\{ \left( \mathbf{S}_{\text{Rx}}^H(\hat{\theta}) \mathbf{R}_{\text{zx}}^{-1} \mathbf{S}_{\text{Rx}}(\hat{\theta}) \right)^{-1} \times \mathbf{S}_{\text{Rx}}^H(\hat{\theta}) \mathbf{R}_{\text{zx}}^{-1} \mathbf{R}_{\text{ex}} \mathbf{S}_{\text{Tx}}(\hat{\theta}) \right\}. \quad (61)$$

A detailed derivation is given in Appendix C. Compared with the solution of $\beta$ given in (35), this optimal solution of $\beta$ will be used to provide a cost function which is only a cost function of $\hat{\theta}$. Here, due to the identifiability of linear system and the independent noise effects, the cost function $\xi_{\text{BBO}}(\hat{\theta})$ will be formed as the sum of the mses of the received signals at Point (B) shown in Fig. 1. This can be expressed as follows:

$$\hat{\theta} = \arg \min_{\theta} \xi_{\text{BBO}}(\theta) \quad (62)$$

where

$$\xi_{\text{BBO}}(\theta) = \frac{1}{L} \left\| X - \mathbf{S}_{\text{Rx}}^H(\theta) \text{diag} \left\{ \beta(\theta) \right\} \mathbf{S}_{\text{Tx}}^H(\theta) \mathbf{M} \right\|^2 \quad \text{Eqn. (61)} \quad (63)$$

Following the estimates of $\hat{\theta}$, the estimates of the corresponding complex path gains $\hat{\beta}$ can be evaluated according to (61).

As shown in (63), with respect to $\hat{\theta}$, the cost function $\xi_{\text{BBO}}(\theta)$ is highly nonlinear with multiple local solutions. As a result, traditional approaches such as gradient-based methods will fail. This highly nonlinear problem can be solved by employing biology-based optimization methods such as genetic algorithms [27], particle swarm optimization [28], or the recently emerged BBO algorithm [21]. The BBO approach uses a mechanism which is analogous to nature’s way of distributing species. It is shown in [21] that, since the cost function [i.e., (62)] proposed in this paper is multimodal and nonseparable, it is more desirable to use the BBO algorithm rather than the other approaches to achieve a better convergence time and accuracy.

C. Complexity Analysis

The difference of the computational complexity of the two proposed methods per iteration mainly depends on the spatial spectrum searching. The computational complexity of the proposed 1-D iterative method is $O(K L_\theta)$, where $L_\theta$ denotes the size of the spatial spectrum. The worst case for the multidimensional optimal method requires a multidimensional search for all the possiblek solutions, which has a computational complexity of $O((1/K!) M_{\text{Rx}}(L_\theta - i + 1)) \approx O(1/10^6)$. Note that this is extremely high compared to $O(K L_\theta)$. This computational complexity is significantly reduced by using the BBO algorithm. The average number of flops in one realization based on the simulation in Section V-A illustrates the advantages of applying BBO in the proposed multidimensional optimization problem. Here, the number of flops is of the order of $O(10^6)$ for the BBO-based multidimensional optimal method, which is ten times the number of flops required by the proposed 1-D approach.

D. Comparison With Other Methods

For convenience, the main expressions relating to the existing methods, which will be compared to the two proposed ones, are given in Table I.

<table>
<thead>
<tr>
<th>Expression of Multitarget Parameter Estimation Methods</th>
<th>LS</th>
<th>Capon</th>
<th>APES</th>
<th>Proposed 1</th>
<th>Proposed 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\beta_{\text{LS}}(\theta)$</td>
<td>$\frac{\mathbf{S}<em>{\text{Rx}}^H(\theta) \mathbf{R}</em>{\text{ex}}^{-1} \mathbf{R}<em>{\text{zx}} \mathbf{S}</em>{\text{Tx}}(\theta)}{N_{\text{Tx}} N_{\text{Rx}}}$</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\beta_{\text{APES}}(\theta)$</td>
<td>$\beta_{\text{APES}}(\theta)$</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\beta_{\text{CAML}}(\theta)$</td>
<td>$\mathbf{S}<em>{\text{Rx}}^H(\theta) \mathbf{R}</em>{\text{zx}}^{-1} \mathbf{R}<em>{\text{ex}} \mathbf{S}</em>{\text{Tx}}(\theta)$</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Proposed 1</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Proposed 2</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

The $(\theta_i, \beta_i)$, $k = 1, \ldots, K$, of the LS and Capon methods are obtained by searching the peaks of the spectra of $|\beta_i(s(\theta_i))|$, $|\beta_{\text{CAML}}(\theta_i)|$, and $|\beta_{\text{APES}}(\theta_i)|$, respectively, with the variant $\theta$. As discussed in [17], the Capon method provides more accurate estimates of the DOAs of targets but worse estimates of the path gains by comparing with the APES method. Thus, to obtain the benefits of both Capon and APES, a combined approach, named CAPES, is proposed in [29]. CAPES first estimates the DOAs of targets utilizing the Capon method and then refines the estimates of the path gains using the APES method. Similarly, the CAML method combines the Capon estimator and the AML method [30]. Thus, the performances of the
TABLE II
SETTING OF PARAMETERS IN BBO ALGORITHM

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Habitat modification probability</td>
<td>1</td>
</tr>
<tr>
<td>Immigration probability bounds per gene</td>
<td>[0, 1]</td>
</tr>
<tr>
<td>Step size for numerical integration of probabilities</td>
<td>1</td>
</tr>
<tr>
<td>Maximum immigration and migration rates for each island</td>
<td>1</td>
</tr>
<tr>
<td>Mutation probability</td>
<td>0.9</td>
</tr>
<tr>
<td>Population size</td>
<td>1000</td>
</tr>
<tr>
<td>Granularity</td>
<td>0.05</td>
</tr>
</tbody>
</table>

CAPES and CAML methods are restricted to the DOA estimation given by Capon, even if they can provide more accurate estimation of the complex path gains than LS, Capon, and APES. It is worth pointing out that the Multiple-Signals-Classification (MUSIC) algorithm is not included in the comparison. The MUSIC algorithm provides only the DOA estimation but no estimation of complex path gains which are related to the range and cross section of the targets. Furthermore, MUSIC provides poorer estimation than the proposed approaches and other existing methods since it estimates the DOAs using only the Rx antenna array and the covariance matrix of the received signals ($\mathbf{R}_{xx}$). It does not utilize the manifold vector of the Tx array as in the proposed approaches and the other existing methods.

V. NUMERICAL EXAMPLES

Consider a MIMO radar system where a uniform linear array with $N_{Tx} = 10$ antennas with half-wavelength spacing is employed for both Tx and Rx arrays. For simplicity but without loss of generality, the relative delays of the targets are assumed negligible. The probing waveforms $m_l(t)$ employ Hadamard codes which have good orthogonality properties, and $P_T$ is the total transmit power. Assume that there are $K = 3$ targets located at $\theta_1 = 60^\circ$, $\theta_2 = 72^\circ$, and $\theta_3 = 79^\circ$ with complex path gains $\beta_1 = 0.7 e^{j1.3}$, $\beta_2 = 0.8 e^{j0.9}$, and $\beta_3 = 0.8 e^{j0.7}$, respectively, which need to be estimated. The received signal is corrupted by a white circularly symmetric complex Gaussian noise with zero mean and variance $\sigma_n^2$. The average root mean square errors (rmse) of the estimated parameters $\hat{\theta}_k$, $\hat{\beta}_k$, and $k = 1, \ldots, K$ are defined as

$$\text{rmse}_\theta = \frac{1}{K} \sum_{k=1}^{K} \mathbb{E} \left\{ |\theta_k - \hat{\theta}_k|^2 \right\}$$

$$\text{rmse}_\beta = \frac{1}{K} \sum_{k=1}^{K} \mathbb{E} \left\{ |\beta_k - \hat{\beta}_k|^2 \right\}$$

and are used to compare the performances of the proposed methods with that of other methods. Moreover, here, the initial receive beamforming matrix $\mathbf{W}_{Rx}$ of the proposed 1-D iterative method is formed according to (56) by using the estimated DOAs and path gains from the CAPES method. For the BBO approach employed in the proposed multidimensional optimal method, the setting of the parameters is given in Table II. The results are all obtained by 1000 Monte Carlo simulations.

A. Performance of Proposed Methods Versus Iterations

First, consider a scenario where the number of snapshots $L = 128$ and $P_T/\sigma_n^2$ equals $0$ dB. Fig. 2(a)-(d) shows the magnitudes of the spatial spectral estimates of $\hat{\beta}_k$, versus $\hat{\theta}_k$, $k = 1, \ldots, K$, obtained in different iterations of the proposed 1-D iterative method in a particular realization. Table III provides the average estimation error of the two proposed algorithms as well as Capon, CAPES, CAML and LS. In these figures, the real $\theta_k$ and $\beta_k$, $k = 1, \ldots, K$, are denoted by the green crosses while the estimated $\hat{\theta}_k$ and $\hat{\beta}_k$, $k = 1, \ldots, K$, are represented via the colored circles. It can be seen that the estimates become closer to the real values as the number of iterations increases for the proposed method.

Since the BBO approach for the multidimensional optimal method provides the estimates in an iterative way as well, Figs. 3 and 4 show the rmse convergence of $\hat{\theta}_k$ and $\hat{\beta}_k$, $k = 1, \ldots, K$, of the proposed two methods, respectively. For comparative purposes, the LS, Capon, CAPES, and CAML methods are represented by the horizontal lines (although they are NOT iterative methods), while the APES method fails to perform since the targets are too close to be resolved.

Comparing with these existing methods, the rmse and rmse of the proposed 1-D iterative method (see Proposed 1 in the figures) decrease significantly after several iterations, which implies the improvement of the accuracy of the estimates provided by the proposed 1-D iterative method. Since the proposed iterative method is an algorithmic iterative procedure, it cannot be analytically proven. However, using 1000 computer simulation studies, 100% success rate of convergence has been shown for the proposed iterative algorithm. Moreover, the “error bars” in Figs. 3 and 4 represent the standard deviation for the rmse and rmse, respectively, which show that the proposed iterative method always outperforms the existing ones.

The rmse and rmse of the proposed multidimensional optimal method (see Proposed 2 in the figures) achieve a lower level than those of the proposed 1-D iterative method. This implies that the multidimensional optimal method provides more accurate estimation than the 1-D iterative method. This is due to the fact that the multidimensional optimal method utilizes the optimal solution of the complex path gain vector $\beta$ for the minimization problem in (26), while the 1-D iterative method utilizes the suboptimal solution of $\beta$ with fixed $\mathbf{W}_{Rx}$ according to (35). Moreover, if the initial estimates of the directions of targets given by Capon are applied in the BBO for the proposed multidimensional method, as the dash-dotted line shown in Figs. 3 and 4, the convergence rates of rmse and rmse are even better than those of the random initialized BBO (shown as the dashed line). From Figs. 3 and 4, it is clear that both the proposed methods outperform the other existing methods as expected.

B. Finite Averaging Effects

Next, with $P_T/\sigma_n^2$ fixed at 0 dB, the variations of rmse and rmse as a factor of the number of snapshots are shown in

$^1$Note that the input SNR at the Rx array is smaller than 0 dB due to the path gains that attenuate the transmit signals.
C. Noise Effects (Variable Levels of $P_T/\sigma_n^2$)

Finally, Figs. 7 and 8 show the effect of $P_T/\sigma_n^2$ on the performance of the proposed methods while $P_T/\sigma_n^2$ is varying. The number of snapshots is fixed at $L = 128$, and all the other parameters are set as before. As expected, the proposed multidimensional optimal method shows its superiority in the parameter estimation than all the other methods throughout the whole range of $P_T/\sigma_n^2$. Meanwhile, the proposed 1-D iterative method outperforms the LS, Capon, CAPES, and CAML methods, particularly when $P_T/\sigma_n^2$ is low. When $P_T/\sigma_n^2$ is high, the Capon, CAPES, and CAML methods tend to reach the similar $\text{rms}_{\theta}$ performance as the proposed 1-D iterative method while

---

**TABLE III**

**AVERAGE ESTIMATION ERROR**

<table>
<thead>
<tr>
<th>Methods</th>
<th>Avg. Error of $\hat{\theta}$</th>
<th>Avg. Error of $\hat{\beta}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Proposed 1</td>
<td>0.27</td>
<td>0.10</td>
</tr>
<tr>
<td>Proposed 2</td>
<td>0.13</td>
<td>0.07</td>
</tr>
<tr>
<td>Capon</td>
<td>0.95</td>
<td>0.40</td>
</tr>
<tr>
<td>CAPES</td>
<td>0.95</td>
<td>0.39</td>
</tr>
<tr>
<td>CAML</td>
<td>0.95</td>
<td>0.37</td>
</tr>
<tr>
<td>LS</td>
<td>1.35</td>
<td>0.52</td>
</tr>
</tbody>
</table>

Figs. 5 and 6, respectively. It is clear that, for the entire range of snapshots, the proposed multidimensional optimal method has the ability to give the best parameter estimation of multiple targets while the proposed 1-D iterative method is worse than the multidimensional optimal one but significantly enhances the accuracy of the estimation given by the rest of the methods, even with a small number of snapshots. These results coincide with the case of fixed $P_T/\sigma_n^2$ and snapshots and the explanation therein.
Fig. 4. $\text{rmse}_\beta$ convergence of the proposed multidimensional optimal and 1-D iterative methods when $P_T/\sigma_n^2$ equals 0 dB and $L$ equals 128 snapshots.

Fig. 5. $\text{rmse}_\theta$ performance versus the number of snapshots when $P_T/\sigma_n^2$ equals 0 dB.

Fig. 6. $\text{rmse}_\beta$ performance versus the number of snapshots when $P_T/\sigma_n^2$ equals 0 dB.

Fig. 7. Performance of $\text{rmse}_\theta$ versus $P_T/\sigma_n^2$ when $L$ equals 128 snapshots.

Fig. 8. Performance of $\text{rmse}_\beta$ versus $P_T/\sigma_n^2$ when $L$ equals 128 snapshots.

the CAPES and CAML methods approach a similar $\text{rmse}_\beta$ as the proposed 1-D iterative method. However, it is inappropriate to assume such a high $P_T/\sigma_n^2$ environment for a radar system since the target returns are seldom hundreds of times stronger than the noise. It is worth noting that, when $P_T/\sigma_n^2$ increases, the $\text{rmse}_\beta$ decreases for all described methods except the Capon method since the estimate of $\theta$ is biased and the estimates of $\beta$ is unbiased in Capon while, for all the other methods, the estimates of both $\theta$ and $\beta$ are unbiased.

VI. CONCLUSION

This paper has focused on multitarget parameter estimation with high mutual interferences among the echoes from the targets which are close together. Inspired by the APES method, an optimization problem which involves the suppression of these interferences for multitarget parameter estimation was introduced. Based on this, the multidimensional optimal and 1-D iterative methods were proposed. The BBO approach was
utilized for the multidimensional optimal method to avoid a multidimensional search. The 1-D iterative method eliminates the need for multidimensional optimization, but the initial estimates are required. The numerical results illustrate that both methods have better performance than other existing methods since they suppress the mutual interferences among the echoes from the targets and the others do not. Moreover, the multidimensional optimal method provides more accurate estimates than the 1-D iterative method. This is because the former method utilizes the optimal solution of the complex path gain while the latter uses the suboptimal solution of the complex path gain for the formulated optimization problem. However, it must be noted that the multidimensional optimal method with BBO has higher computational complexity than the 1-D iterative one, particularly when the number of targets is large. Hence, there is a tradeoff when choosing these two methods in practice.

APPENDIX A

PROOF OF LEMMAS 1 AND 2

Let \( a_{ij} \) and \( c_{ij} \) denote the \((i,j)\)th elements of \( A \in \mathbb{C}^{M \times N} \), \( \mathbb{D} \in \mathbb{C}^{N \times N} \), and \( C \in \mathbb{C}^{N \times N} \), respectively. Let \( b_j \) denote the \( j\)th element of \( b \in \mathbb{C}^{N \times 1} \). Then

\[
[A \text{diag}(b)C]_{i,j} = \sum_{k=1}^{K} a_{ij} b_j c_{kj} = \sum_{k=1}^{K} a_{ij} c_{kj} b_j = [(A \circ C^T)b]_{i,j}.
\]

Thus, it is straightforward to obtain the following result:

\[
\text{diag} \{A \text{diag}(b)C\} = (A \circ C^T)b
\]

which is given in Lemma 1.

By Lemma 1, the derivative of \( \text{tr} \{ \mathbb{D} \text{diag}(b) \} \) with respect to \( b \) can be rewritten as

\[
\frac{\partial}{\partial b} \text{tr} \{ \mathbb{D} \text{diag}(b) \} = \frac{\partial}{\partial b} \left[ 1_N^T \text{diag} \{ \mathbb{D} \text{diag}(b) \} 1_N \right] = \frac{\partial}{\partial b} \left[ 1_N^T (\mathbb{D} \circ 1_N)b \right] = \frac{\partial}{\partial b} \left[ (\text{diag} \{ \mathbb{D} \})^T b \right] = \text{diag} \{ \mathbb{D} \}
\]

which is the result in Lemma 2.

APPENDIX B

DERIVATION OF \( \bar{W}_{RX} \)—(56)

First, according to the definitions in (44) and (45), replacing \( \bar{S}_{RX}(\theta) \) and \( \bar{R}_{xx} \) of the term \( (S^H_{RX}(\theta)\bar{R}_{xx}^{-1}S_{RX}(\theta))^{-1} \) in (55) yields

\[
\left( S^H_{RX}(\theta)\bar{R}_{xx}^{-1}S_{RX}(\theta) \right)^{-1} = \left[ (I_K \otimes S^H_{RX}(\theta)) (I_K \otimes \bar{R}_{xx}^{-1}) (I_K \otimes S_{RX}(\theta)) \right]^{-1} = \left[ I_K \otimes \left( S^H_{RX}(\theta)\bar{R}_{xx}^{-1}S_{RX}(\theta) \right) \right]^{-1} = \left( I_K \otimes S^H_{RX}(\theta) \right) \bar{R}_{xx}^{-1}S_{RX}(\theta)^{-1}.
\]

Then, by using (40), the term \( \bar{S}_{RX}(\theta)\bar{R}_{xx}^{-1}u \) in (55) can be rewritten as

\[
\bar{S}_{RX}(\theta)\bar{R}_{xx}^{-1}u = \left( I_K \otimes \bar{S}_{RX}^H(\theta) \bar{R}_{xx}^{-1} \right) \text{vec} \{ \bar{R}_{xx} S_{TX}(\theta) \text{diag} \{ \bar{\beta}^T \} \} = \text{vec} \{ S^H_{RX}(\theta) \bar{R}_{xx}^{-1} \bar{R}_{xx} S_{TX}(\theta) \text{diag} \{ \bar{\beta}^T \} \}.
\]

Thus, the vectorized weight matrix \( \bar{W}_{RX} \) in (55) can be rewritten as

\[
\text{vec} \{ \bar{W}_{RX} \} = \left( I_K \otimes \bar{R}_{xx}^{-1} \right) \text{vec} \{ \bar{R}_{xx} S_{TX}(\theta) \text{diag} \{ \bar{\beta}^T \} \}
\]

\[
- \left[ I_K \otimes \left( \bar{R}_{xx}^{-1} \left( S^H_{RX}(\theta) \bar{R}_{xx}^{-1} S_{RX}(\theta) \right)^{-1} \right) \right] \text{vec} \{ S^H_{RX}(\theta) \bar{R}_{xx}^{-1} \bar{R}_{xx} S_{RX}(\theta) \text{diag} \{ \bar{\beta}^T \} - I_K \}
\]

\[
= \text{vec} \left\{ \bar{R}_{xx}^{-1} \bar{R}_{xx} S_{RX}(\theta) \text{diag} \{ \bar{\beta}^T \} \right\}
\]

\[
- \text{vec} \left\{ \bar{R}_{xx}^{-1} \bar{R}_{xx} S_{TX}(\theta) \text{diag} \{ \bar{\beta}^T \} - I_K \right\}.
\]

Afterward, removing the vectorization operators yields

\[
\bar{W}_{RX} = \bar{R}_{xx}^{-1} \bar{R}_{xx} S_{TX}(\theta) \text{diag} \{ \bar{\beta}^T \}
\]

\[
- \bar{R}_{xx}^{-1} \bar{R}_{xx} S_{RX}(\theta) \left( \bar{R}_{xx}^{-1} \bar{R}_{xx} S_{RX}(\theta) \right)^{-1}
\]

\[
- \bar{R}_{xx}^{-1} \bar{R}_{xx} S_{TX}(\theta) \left( \bar{R}_{xx}^{-1} \bar{R}_{xx} S_{TX}(\theta) \right)^{-1}
\]

\[
\left( S^H_{RX}(\theta) \bar{R}_{xx}^{-1} \bar{R}_{xx} S_{TX}(\theta) \text{diag} \{ \bar{\beta}^T \} - I_K \right)
\]

and rearranging the terms of the equation above gives the result of \( \bar{W}_{RX} \) in (56).

APPENDIX C

DERIVATION OF \( \beta(\theta) \)—(61)

Next, based on Lemma 1, inserting \( \bar{W}_{RX} \) given in (56) into (35) yields

\[
N_{TX} \beta = \text{diag} \{ \bar{W}_{RX}^H \bar{R}_{xx} S_{TX}(\theta) \}
\]

\[
= \text{diag} \left\{ \bar{W}_{RX}^H \bar{R}_{xx}^{-1} \bar{R}_{xx} S_{TX}(\theta) \right\} \cdot \text{diag} \left\{ \left( \bar{S}_{RX}(\theta) \bar{R}_{xx}^{-1} S_{RX}(\theta) \right)^{-1} \times S_{TX}(\theta) \right\}
\]

\[
- \text{diag} \{ \bar{W}_{RX}^H \bar{R}_{xx}^{-1} \bar{R}_{xx} S_{TX}(\theta) \}
\]

\[
- \text{diag} \left\{ \left( \bar{S}_{RX}(\theta) \bar{R}_{xx}^{-1} S_{RX}(\theta) \right)^{-1} \times S_{TX}(\theta) \right\}
\]

\[
= \left[ I_K \otimes \left( \bar{S}_{TX}(\theta) \bar{R}_{xx}^{-1} \bar{R}_{xx} S_{TX}(\theta) \right) \right] \beta
\]

\[
+ \text{diag} \left\{ \left( \bar{S}_{RX}(\theta) \bar{R}_{xx}^{-1} S_{RX}(\theta) \right)^{-1} \right\}
\]
Then, rearranging the terms gives the solution of $\beta(\theta)$ in (61).

**REFERENCES**


Kai Luo (S’11) received the B.Eng. degree in telecommunications engineering from the Huazhong University of Science and Technology, Wuhan, China, in 2006. He joined the Department of Electrical and Electronic Engineering at Imperial College London, London, U.K., in 2007, where he is currently working toward the Ph.D. degree, under the supervision of Prof. Manikas, in the area of array signal processing and its applications to MIMO radar and wireless communications.

He is sponsored by the U.K.-China Scholarships for Excellence programme (U.K. and Chinese governments). His research interests include array signal processing, radar signal processing, convex optimization, adaptive filtering, compressive sensing, remote sensing, and MIMO communications.

Athanasios Manikas (SM’02) received the D.I.C Diploma from the Imperial College London, U.K., and the Ph.D. degree from the University of London, U.K., in 1988 and was appointed Lecturer at Imperial College London on the same year.

He currently holds the Chair in Communications and Array Processing in the Department of Electrical and Electronic Engineering, Imperial College London. He is the Technical Lead of the University Defense Research Centre in Signal Processing, which is supported by the Ministry of Defence (U.K.) and Engineering and Physical Sciences Research Council. He is on the editorial board of the Institute of Engineering and Technology (IET) Proceedings in Signal Processing and is the Editor of the IET press research book series on Communications and Signal Processing. He has published an extensive set of journal and conference papers in the area of digital communications and array signal processing and is the author of a book (monograph) entitled "Differential Geometry in Array Processing." In addition, he has held a number of research consultancies for the European Union, industry, and government organizations and has had various technical chairs at international conferences.

Prof. Manikas has served as an Expert Witness in the High Court of Justice (U.K.) and as a member of the Royal Society’s International Fellowship Committee. He is leading a strong group of researchers at Imperial College and has successfully supervised 35 PhDs and more than 150 Master project students. He is a Fellow of IET and a Chartered Engineer.