A NEW GENERAL GLOBAL ARRAY CALIBRATION METHOD

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ABSTRACT

In this paper, a new method is proposed for calibrating a planar array of general geometry with respect to errors in location, phase and gain, even when all errors are present simultaneously (global calibration). The method requires the use of three pilot sources, which operate one at a time and from at least two different azimuth angles. Since the sources need not operate simultaneously, one only pilot source may be used from three different directions. The method assumes that the electrical errors (phase and gain errors) are not direction de-pendent. Simulation results that support the performance of the proposed method are also presented.

1. INTRODUCTION

The practical applications of Signal-Subspace techniques have been limited due to the fact that calibration errors relating to the electrical and/or geometrical characteristics of the array, can severely degrade their performance. Figure 1 illustrates the performance of MuSiC algorithm for a 7-element, nominally linear array, and three uncorrelated sources in the far-field with azimuths of 30°, 35° and 140°, when the array is calibrated and when it is not (electrical or geometrical errors). It is obvious that when the array is calibrated, the directions-of-arrival (DOA's) of the impinging signals are correctly identified by the means of three deep nulls in the spatial spectrum. When calibration errors exist, it is not possible to estimate any more the DOA's of the impinging sources.

Previous attempts to solve the calibration problem are either applicable in particular array geometries [1, 2, 3, 4, 5], or tackle only geometrical errors [6, 7, 8, 9, 10, 11, 12, 13], or only electrical errors [14, 15]. In the field of global calibration, Clements and Lo, [16], presented a recursive algorithm that considers that the

array errors are Gaussian random variables with known means and variances. Wang and Cadzow, [17], presented an iterative algorithm suitable only for small perturbations and high SNR. Finally, Koerber and Fuhrmann, [18], use a Fourier series parameterization of the array manifold and propose a ML solution that employs a large number of sources and uses a QR decomposition method to handle the gain scaling problem.

Figure 1: Performance of MuSiC algorithm in the presence of errors in the nominal values of the characteristics of the elements of a 7-element linear array.

In Section 2, the mathematical modelling of the locational and electrical errors will be described and the way these errors affect the data covariance matrix will be explained. In Section 3, the proposed calibration method will be presented. Then, in Section 4, simulation results will be used to demonstrate the performance of the proposed method. Finally, in Section 5 the paper will be concluded.

2. MATHEMATICAL MODEL

The covariance matrix of the signals due to one pilot source of unity power, received by an array of N elements can be modelled as follows:

$$\hat{\mathbb{R}}_{xx} = S \cdot S^H + \mathbb{R}_{nn} \quad (1)$$

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where S is the Source Position Vector (SPV) or Manifold Vector and $\mathbb{R}_{nn}$ is the noise covariance matrix. Without any loss of generality, it can be assumed that the noise is thermal, isotropic and uncorrelated from element to element. Thus, $\mathbb{R}_{nn} = \sigma^2 \mathbb{I}$, where σ^2 is the noise power and $\mathbb{I}$ is the identity matrix. Even if the noise is not isotropic, as long as its distribution is known, it can be transformed to appear isotropic by suitable pre-processing of the received samples of the signal. The SPV is a mapping of the characteristics of the transmitting source:

- azimuth and elevation angle,
 - frequency of the transmitted signal, and
 - source range, for spherical wave-propagation
- and the characteristics of the array:
- geometry,
 - gain, and phase

on to a vector in an N-dimensional complex space, and is defined as:

$$S = \gamma \odot \psi \odot \mathbf{a} \quad (2)$$

where:

$$\begin{aligned} \gamma &= [\gamma_1 \ \dots \ \gamma_N]^T, \\ \psi &= e^{j\phi}, \text{ with } \phi = [\phi_1 \ \dots \ \phi_N]^T, \text{ and} \\ \mathbf{a} &= e^{-j\mathbf{r}^T \mathbf{k}}, \text{ with } \mathbf{r} = [x_1 \ \dots \ x_N]. \end{aligned}$$

Note that γ_i , ϕ_i and x_i , $i = 1 \cdot \dots \cdot N$, are respectively the gain, the phase and the coordinates of the i^{th} element, and $\mathbf{k}$ is the wavelength vector associated with the pilot source. In general, the wavelength vector, $\mathbf{k}$, is defined as:

$$\mathbf{k} = \mathbf{k}(\theta, \phi, \lambda) = \frac{2\pi}{\lambda} \cdot \begin{bmatrix} \cos(\theta) \cdot \cos(\phi) \\ \sin(\theta) \cdot \cos(\phi) \\ \sin(\phi) \end{bmatrix}$$

where θ and ϕ are the azimuth and the elevation angles of the pilot source, and λ is the wavelength of the transmitted signal.

Let the k^{th} element of the array, $k=1 \cdot \dots \cdot N$, exhibiting a gain error $\tilde{\gamma}_k$, a phase error $\tilde{\phi}_k$, and a location error $\tilde{\mathbf{r}}_k$. The true SPV, $\hat{S}$, which takes into account errors in the nominal values, can be expressed in terms of the nominal SPV, S , as:

$$\hat{S} = S + S \odot \left(\frac{\gamma + \tilde{\gamma}}{\gamma} \odot \tilde{\psi} \odot \tilde{\mathbf{a}} - \mathbf{1} \right). \quad (3)$$

where

$$\begin{aligned} \tilde{\gamma} &= [\tilde{\gamma}_1 \ \dots \ \tilde{\gamma}_N]^T, \\ \tilde{\psi} &= e^{j\tilde{\phi}}, \text{ with } \tilde{\phi} = [\tilde{\phi}_1 \ \dots \ \tilde{\phi}_N]^T, \text{ and} \\ \tilde{\mathbf{a}} &= e^{-j\tilde{\mathbf{r}}^T \mathbf{k}}, \text{ with} \\ \tilde{\mathbf{r}} &= [\tilde{x}_1 \ \dots \ \tilde{x}_N] = [\tilde{x}, \ \tilde{y}, \ \tilde{z}]^T. \end{aligned}$$

Note that $\tilde{\gamma}_i$, $\tilde{\phi}_i$ and $\tilde{x}_i$, $i = 1 \cdot \dots \cdot N$, are respectively the errors in gain, in phase and in the coordinates of the

i^{th} element. Furthermore, $\tilde{x}$, $\tilde{y}$ and $\tilde{z}$ correspond to the errors in the x, y and z coordinates. The deviation, $\tilde{S}$, of the true SPV from its nominal value can be expressed as:

$$\tilde{S} = S \odot \left(\frac{\gamma + \tilde{\gamma}}{\gamma} \odot \tilde{\psi} \odot \tilde{\mathbf{a}} - \mathbf{1} \right) = \tilde{\Delta} \cdot S \quad (4)$$

with

$$\tilde{\Delta} = \text{diag} \left(\frac{\gamma + \tilde{\gamma}}{\gamma} \odot \tilde{\psi} \odot \tilde{\mathbf{a}} - \mathbf{1} \right).$$

Thus, in the presence of errors regarding the electrical and geometrical characteristics of the array, the data covariance matrix becomes:

$$\begin{aligned} \hat{\mathbb{R}}_{xx} &= (S + \tilde{S}) \cdot (S + \tilde{S})^H + \sigma^2 \mathbb{I} = \\ &= (\mathbb{I} + \tilde{\Delta}) \cdot S \cdot S^H \cdot (\mathbb{I} + \tilde{\Delta})^H + \sigma^2 \mathbb{I}. \end{aligned} \quad (5)$$

Since Signal Subspace type techniques assume for the covariance matrix the model defined in Equation 1 rather than Equation 5, they do not take into consideration any errors relating to the array characteristics, that is $\tilde{S}$. Thus, their performance is unacceptable in the presence of such errors, as it was demonstrated in Figure 1.

3. GLOBAL CALIBRATION

Consider an uncalibrated array of N elements, which operates in the presence of one pilot source. The uncalibrated array suffers from both geometrical and electrical uncertainties. If γ and ψ are the nominal values for the gain and the phase of the array and $\mathbf{a}$ is the nominal isotropic SPV which

corresponds to the pilot source then a matrix $\mathbb{O}$:

$$\mathbb{O} = \text{diag}(\psi \odot \mathbf{a}) \quad (6)$$

can be defined. Note that $\text{diag}(\text{matrix})$ denotes a column vector and $\text{diag}(\text{vector})$ denotes a diagonal matrix. By pre- and post-processing the data

covariance matrix, $\hat{\mathbb{R}}_{xx}$, by the above defined matrix

$\mathbb{O}$, a new matrix $\mathbb{O}^H \cdot \hat{\mathbb{R}}_{xx} \cdot \mathbb{O}$ is established. It can be shown that this known matrix can be written as a function of the array uncertainties as follows:

$$\mathbb{O}^H \cdot \hat{\mathbb{R}}_{xx} \cdot \mathbb{O} = \text{diag}(\mathbf{U}) \cdot \text{diag}(\mathbf{U})^H + \sigma^2 \mathbb{I}. \quad (7)$$

where:

$$\mathbf{U} = (\mathbb{I} + \tilde{\Delta}) \cdot \text{diag}(\gamma) = \mathbf{\Gamma} \cdot \tilde{\Psi} \cdot \tilde{\mathbf{A}} \quad (8)$$

and

$$\mathbf{\Gamma} = \text{diag}(\gamma + \tilde{\gamma}), \quad (9)$$

$$\tilde{\Psi} = \text{diag}(\tilde{\psi}), \text{ and} \quad (10)$$

$$\tilde{\mathbf{A}} = \text{diag}(\tilde{\mathbf{a}}). \quad (11)$$

It is common to regard one element as a reference and to assume that its characteristics are known. For simplicity, the first element of the array is taken as the reference element. This translates to the fact that the element

$\mathbb{U}(1,1)$ of the matrix $\mathbb{U}$ will be equal to the known gain, say γ_1 , of the reference element. Thus, $\mathbb{U}$ becomes:

$$\mathbb{U} = \text{diag}\left(\begin{bmatrix} \gamma_1 \\ \mathbf{u} \end{bmatrix}\right) \Rightarrow \text{diag}(\mathbb{U}) = \begin{bmatrix} \gamma_1 \\ \mathbf{u} \end{bmatrix} \quad (12)$$

where the vector $\mathbf{u}$ corresponds to the rest $N-1$ elements of the matrix $\mathbb{U}$. Using the above argument Equation 7 becomes:

$$\mathbb{O}^H \cdot \hat{\mathbb{R}}_{xx} \cdot \mathbb{O} = \begin{bmatrix} \gamma_1^2 + \sigma^2 & \gamma_1 \cdot \mathbf{u}^H \\ \gamma_1 \cdot \mathbf{u} & \mathbf{u} \cdot \mathbf{u}^H + \sigma^2 \end{bmatrix}. \quad (13)$$

Equations 12 and 13 can be used to calculate the matrix $\mathbb{U}$, as well as, the noise power σ^2 .

From the above discussion, it is clear that using one pilot source and the corresponding data covariance matrix, a matrix $\mathbb{U}$, which is a function of the array uncertainties can be estimated. If this matrix is estimated for two different pilot sources, then two matrices, say $\mathbb{U}_1$ and $\mathbb{U}_2$, are formed. As long as the electrical uncertainties are non-directional, matrices $\mathbb{U}_1$ and $\mathbb{U}_2$ can be expressed, using Equation 8, as follows:

$$\mathbb{U}_1 = \mathbf{\Gamma}_1 \cdot \tilde{\mathbf{\Psi}} \cdot \tilde{\mathbf{A}}_1 \quad (14)$$

with $\tilde{\mathbf{A}}_1 = \text{diag}(e^{-j\tilde{\mathbf{r}}^T \cdot \mathbf{k}_1})$, $\mathbf{\Gamma}_1 = \text{diag}(\gamma_1 + \tilde{\gamma})$, and

$$\mathbb{U}_2 = \mathbf{\Gamma}_2 \cdot \tilde{\mathbf{\Psi}} \cdot \tilde{\mathbf{A}}_2 \quad (15)$$

with $\tilde{\mathbf{A}}_2 = \text{diag}(e^{-j\tilde{\mathbf{r}}^T \cdot \mathbf{k}_2})$ and $\mathbf{\Gamma}_2 = \text{diag}(\gamma_2 + \tilde{\gamma})$.

Since the electrical uncertainties are non-directional, matrix $\tilde{\mathbf{\Psi}}$ is common for both $\mathbb{U}_1$ and $\mathbb{U}_2$. Note that matrices $\tilde{\mathbf{A}}_1$, $\mathbf{\Gamma}_1$, and $\tilde{\mathbf{A}}_2$, $\mathbf{\Gamma}_2$ depend on the direction of the two pilot sources. Matrices $\tilde{\mathbf{A}}_1$ and $\tilde{\mathbf{A}}_2$ affect the phase component of $\mathbb{U}_1$ and $\mathbb{U}_2$, whereas matrices $\mathbf{\Gamma}_1$ and $\mathbf{\Gamma}_2$ affect the amplitude component of $\mathbb{U}_1$ and $\mathbb{U}_2$. Using Equations 14 and 15, it can be proved that:

$$\angle(\mathbb{U}_2^{-1} \cdot \mathbb{U}_1) = -\text{diag}\left(\tilde{\mathbf{r}}^T \cdot (\mathbf{k}_1 - \mathbf{k}_2)\right) \quad (16)$$

which implies:

$$\tilde{\mathbf{r}}^T \cdot (\mathbf{k}_1 - \mathbf{k}_2) = -\text{diag}(\angle(\mathbb{U}_2^{-1} \cdot \mathbb{U}_1)). \quad (17)$$

Note that $\angle(\cdot)$ represents the angle of the argument. By writing $\mathbf{k}_1 - \mathbf{k}_2$ as:

$$\mathbf{k}_1 - \mathbf{k}_2 = [K_{11} \quad K_{21} \quad K_{31}]^T$$

and using the locational errors in the x, y and z coordinates, Equation 17 can be rewritten in the following form:

$$K_{11}\tilde{x} + K_{21}\tilde{y} + K_{31}\tilde{z} = -\text{diag}\left(\angle(\mathbb{U}_2^{-1} \cdot \mathbb{U}_1)\right). \quad (18)$$

If the array is planar ($\tilde{z} = \tilde{\Omega}$), Equation 18 simplifies to:

$$K_{11}\tilde{x} + K_{21}\tilde{y} = -\text{diag}\left(\angle(\mathbb{U}_2^{-1} \cdot \mathbb{U}_1)\right). \quad (19)$$

Equation 19 describes a set of N equations with $2 \cdot N$ unknowns $(\tilde{x}, \tilde{y})$. Therefore, another set of N equations is needed for a unique solution. Using another pilot source, matrix $\mathbb{U}_3$ can be calculated, and thus a new set of N equations is defined. That is:

$$K_{12}\tilde{x} + K_{22}\tilde{y} = -\text{diag}\left(\angle(\mathbb{U}_3^{-1} \cdot \mathbb{U}_1)\right) \quad (20)$$

with $[K_{12} \quad K_{22} \quad K_{32}]^T = \mathbf{k}_1 - \mathbf{k}_3$.

The solution to Equations 19 and 20 is:

$$\begin{aligned} \tilde{\mathbf{y}} = & -\frac{K_{11}}{K_{11}K_{22} - K_{12}K_{21}} \cdot \text{diag}\left(\angle(\mathbb{U}_3^{-1} \cdot \mathbb{U}_1)\right) + \\ & + \frac{K_{12}}{K_{11}K_{22} - K_{12}K_{21}} \cdot \text{diag}\left(\angle(\mathbb{U}_2^{-1} \cdot \mathbb{U}_1)\right) \end{aligned} \quad (21)$$

$$\tilde{x} = -\frac{1}{K_{11}} \cdot \text{diag}\left(\angle(\mathbb{U}_2^{-1} \cdot \mathbb{U}_1)\right) - \frac{K_{21}}{K_{11}} \cdot \tilde{\mathbf{y}}.$$

Note that in order Equations 19 and 20 to be independent, it is required that at least two out of the three pilot sources, to have different azimuth angles. Once vectors $\tilde{\mathbf{x}}$ and $\tilde{\mathbf{y}}$ have been estimated using Equation

21, then the locational error matrix, $\tilde{\mathbf{r}}$, for a planar array can be formed as:

$$\tilde{\mathbf{r}} = [\tilde{x}, \tilde{y}, \tilde{\Omega}]^T. \quad (22)$$

It remains now only to estimate the gain and the phase of the array elements. This can be done using one of the three matrices $\mathbb{U}_1$, $\mathbb{U}_2$ or $\mathbb{U}_3$. If, for instance, matrix $\mathbb{U}_1$ is used then from Equation 8, it is:

$$\mathbb{U}_1 = \mathbf{\Gamma}_1 \cdot \tilde{\mathbf{\Psi}} \cdot \tilde{\mathbf{A}}_1. \quad (23)$$

The real matrix $\mathbf{\Gamma}_1$ can be calculated by taking into account that matrices $\tilde{\mathbf{\Psi}}$ and $\tilde{\mathbf{A}}_1$ are complex matrices with unity amplitude. This implies that:

$$\begin{aligned} \mathbf{\Gamma}_1 = \text{abs}(\mathbb{U}_1) & \Rightarrow \tilde{\gamma} + \gamma_1 = \text{diag}(\text{abs}(\mathbb{U}_1)) \Rightarrow \\ \tilde{\gamma} & = \text{diag}(\text{abs}(\mathbb{U}_1)) - \gamma_1. \end{aligned} \quad (24)$$

If Matrix $\tilde{\mathbf{A}}_1$ is estimated using Equation 11 then for matrix $\tilde{\mathbf{\Psi}}$ it is:

$$\begin{aligned} \tilde{\mathbf{\Psi}} & = \mathbf{\Gamma}_1^{-1} \cdot \mathbb{U}_1 \cdot \tilde{\mathbf{A}}_1^{-1} \Rightarrow \\ & \Rightarrow e^{j\tilde{\phi}} = \text{diag}(\mathbf{\Gamma}_1^{-1} \cdot \mathbb{U}_1 \cdot \tilde{\mathbf{A}}_1^{-1}) \Rightarrow \\ \tilde{\phi} & = -j \cdot \log(\text{diag}(\mathbf{\Gamma}_1^{-1} \cdot \mathbb{U}_1 \cdot \tilde{\mathbf{A}}_1^{-1})). \end{aligned} \quad (25)$$

Thus, using Equations 21, 22, 24 and 25, an array of general geometry can be calibrated with respect to location, phase and gain errors, all being present simultaneously.

It is straight forward to generalize the method of an array in a 3-D space. In that case, one more source is required that should be placed in a different azimuth from the previous pilot sources. Calibration of single type errors can be seen as a special case of the proposed method.

4. SIMULATION RESULTS

The proposed method has been tested for different configurations, including linear and circular arrays and performed satisfactorily. For instance, for a 7-element uncalibrated array, Table 1 and Table 2 describe the

errors with respect to location, phase and gain, before and after the application of the proposed method.

5. CONCLUSIONS

In this paper the calibration problem in Super-resolution Direction Finding is addressed. A new method has been proposed that performs global calibration of a planar array of general geometry with respect to location, phase and gain simultaneously. The method employs three pilot sources with two at least having different azimuth angles. The sources operate one at a time, thus, one may use one only pilot source and then move this source to two other locations. Calibration with respect to single errors (e.g. location or phase or gain) can be treated as a special case of the general proposed method.

in $\lambda/2$	Initial location errors		Final location errors (*10 ⁻¹⁴)	
	x-coord	y-coord	x-coord	y-coord
element 2	-0.1428	-0.0863	0.0666	-0.8438
element 3	0.0364	0.0913	0.0222	-0.2665
element 4	0.0340	-0.1370	-0.0666	0.7938
element 5	-0.1361	0.1951	-0.0444	0.4607
element 6	0.0861	0.2352	0.1055	-1.1768
element 7	-0.0526	0.0766	-0.0111	0.1776

Table 1: Locational calibration errors using the method presented in this report.

	Initial phase errors	Final phase errors (*10 ⁻¹⁴)	Initial gain errors	Final gain errors (*10 ⁻¹⁴)
element 2	-0.0680	1.8638	0.0406	0.0000
element 3	0.0341	-0.6092	-0.0423	0.0000
element 4	0.0070	1.7565	0.0846	0.0222
element 5	0.0403	1.0117	-0.0688	-0.0111
element 6	-0.0732	-2.6132	-0.0337	0.0000
element 7	-0.0068	0.3988	0.0650	-0.0222

Table 2: Phase and gain calibration errors using the method presented in this report.

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