

Preface

During the past few decades, there has been significant research into sensor array signal processing, culminating in the development of superresolution array processing, which asymptotically exhibits infinite resolution capabilities.

Array processing has an enormous set of applications and has recently experienced an explosive interest due to the realization that arrays have a major role to play in the development of future communication systems, wireless computing and biomedicine (bio-array processing).

However, the ‘heart’ of any application is the structure of the employed array of sensors and this is completely characterized [1] by the *array manifold*. The *array manifold* is a fundamental concept and is defined as the locus of all the response vectors of the array over the feasible set of source/signal parameters. In view of the nature of the array manifold and its significance in the area of array processing and array communications, the role of differential geometry as the most particularly appropriate analysis tool, cannot be over-emphasized.

Differential geometry is a branch of mathematics concerned with the application of differential calculus for the investigation of the properties of geometric objects (curves, surfaces, etc.) referred to, collectively, as ‘manifolds’. This is a vast subject area with numerous abstract definitions, theorems, notations and rigorous formal proofs [2][3] and is mainly confined to the investigation of the geometrical properties of manifolds in three-dimensional Euclidean space \mathcal{R}^3 and in *real* spaces of higher dimension.

However, the array manifolds are embedded not in real, but in N -dimensional complex space (where N is the number of sensors). There-

fore, by extending the theoretical framework of \mathcal{R}^3 to complex spaces, the underlying and under-pinning objective of this book is to present a summary of those results of differential geometry which are exploitable and of practical interest in the study of linear, planar and three dimensional array geometries.

Thanassis Manikas - London 2003

Acknowledgment

This book is based on a number of publications (presented under a unified framework) which I had over the past few years with some of my former research students. These are Dr. J. Dacos, Dr. R. Karimi, Dr. C. Proukakis, Dr. N. Dowlut, Dr. A. Alexiou, Dr. V. Lefkadites, Dr. A. Sleiman. As their teacher and supervisor, I wish to express my pleasure in having had the opportunity in working with them and learning from them too.

I am indebted to my research associates Naveendra and Jason Ng as well as to my MSc student Vincent Chan for reading the manuscript and for making constructive suggestions on how to improve the presentation material.

Furthermore, the assistance of Dr. P. Wilkinson is greatly appreciated.

Keywords for the book:

linear arrays, non-linear arrays, planar arrays, array manifolds, differential geometry, array design, array ambiguities, array bounds, resolution and detection.

Contents

Preface	vii
Chapter 1 Introduction	1
1.1 Nomenclature	4
1.2 Main Abbreviations	5
1.3 Array of Sensors - Environment	5
1.4 Pictorial Notation	9
1.4.1 Spaces/subspaces	9
1.4.2 Projection Operator	9
1.5 Principal Symbols	11
1.6 Modelling the Array Signal Vector and Array Manifold	12
1.7 Significance of Array Manifolds	18
1.8 An Outline of the Book	19
Chapter 2 Differential Geometry of Array Manifold Curves	23
2.1 Manifold Curve Representation - Basic Concepts	23
2.2 Curvatures and Coordinate Vectors in \mathfrak{C}^N	26
2.2.1 Number of Curvatures and Symmetricity in Linear Arrays	27
2.2.2 'Moving Frame' and Frame Matrix	29
2.2.3 Frame Matrix and Curvatures	31
2.2.4 Narrow and Wide Sense Orthogonality	33
2.3 'Hyperhelical' Manifold Curves	34
2.3.1 Deductions from Theorem 2.1	39
2.3.2 Evaluating the Curvatures of Uniform Linear Array Man- ifolds	40

2.4	The Manifold Length and Number of Windings (or Half Windings)	43
2.5	The Concept of ‘Inclination’ of the Manifold	45
2.6	The Manifold-Radii Vector	47
2.7	Appendices	54
2.7.1	Proof of Eq. (2.24)	54
2.7.2	Proof of Theorem 2.1	56
Chapter 3 Differential Geometry of Array Manifold Surfaces		61
3.1	Manifold Metric	63
3.2	The First Fundamental Form	65
3.3	Christoffel Symbol Matrices	66
3.4	Intrinsic Geometry of a Surface	67
3.4.1	Gaussian Curvature	69
3.4.2	Curves on a Manifold Surface: Geodesic Curvature . . .	71
3.4.2.1	Arc Length	72
3.4.2.2	The Concept of Geodcity	72
3.4.3	Geodesic Curvature	73
3.5	The Concept of ‘Development’	75
3.6	Summary	76
3.7	Appendices	78
3.7.1	Proof of Eq. (3.36) - Geodesic Curvature	78
Chapter 4 Non-Linear Arrays: (θ, ϕ)-Parametrization of Array Manifold Surfaces		81
4.1	Manifold Metric and Christoffel Symbols	82
4.2	3D-grid Arrays of Omnidirectional Sensors	84
4.3	Planar Arrays of Omnidirectional Sensors	84
4.4	Families of θ and ϕ Curves on the Manifold Surface	87
4.5	‘Development’ of Non-linear Array Geometries	92
4.6	Summary	98
4.7	Appendices	99
4.7.1	Proof that the Gaussian Curvature of an Omnidirectional Sensor Planar Array Manifold is Zero	99
4.7.2	Proof of the Expression of $\det \mathbb{G}$ for Planar Arrays in Table 4.2	100
4.7.3	Proof of ‘Development’ Theorem 4.6:	102
Chapter 5 Non-Linear Arrays: (α, β)-Parametrization		105

5.1	Mapping from the (θ, ϕ) Parameter Space to Cone-angle Parameter Space	105
5.2	Manifold Vector in Terms of a Cone-Angle	109
5.3	Intrinsic Geometry of the Array Manifold Based on Cone-Angle Parametrization	110
5.4	Defining the Families of α and β Parameter Curves	113
5.5	Properties of α and β Parameter Curves	114
5.5.1	Geodesicity	114
5.5.2	Length of Parameter Curves	115
5.5.3	Shape of α - and β -curves	116
5.6	'Development' of α and β Parameter Curves	120
Chapter 6 Array Ambiguities		123
6.1	Classification of ambiguities	125
6.2	The concept of an ambiguous generator set	128
6.3	Partitioning the Array Manifold Curve into Segments of Equal Length	132
6.3.1	Calculation of Ambiguous Generator Sets of Linear (or ELA) Array Geometries	142
6.4	Representative Examples	143
6.5	Handling Ambiguities in Planar Arrays	147
6.5.1	Ambiguities on ϕ -curves	148
6.5.2	Ambiguities on α -curves/ β -curves	151
6.5.3	Some comments on planar arrays.	159
6.5.4	Ambiguous Generator Lines	163
6.6	Ambiguities and Manifold Length	165
6.7	Appendices	169
6.7.1	Proof of Theorem 6.1	169
Chapter 7 More on Ambiguities: Symmetrical Arrays		171
7.1	Characteristic Points on the Array Manifold.	174
7.2	Symmetry and Non-Uniform Partitions of Hyperhelices	177
7.3	Ambiguities of Rank- $(N - 1)$ and Array Pattern	182
7.4	Planar Arrays	185
7.5	Ambiguity-Free Field-Of-View	187
7.6	Conclusions	189
Chapter 8 Array Bounds		191

8.1	Circular Approximation of an Array Manifold	191
8.2	Accuracy and the Cramer Rao Lower Bound	196
8.2.1	Single Emitter CRB in Terms of Manifold's Differential Geometry	197
8.2.2	Two emitter CRB in terms of principal curvature	200
8.2.2.1	Elevation Dependence of Two Emitters' CRB	204
8.2.2.2	Azimuth Dependence of Two Emitters' CRB .	205
8.3	'Detection' and 'Resolution' Thresholds	206
8.3.1	Estimating the Detection Threshold	209
8.3.2	Estimating the Resolution Threshold	212
8.4	Modelling of the Uncertainty Sphere	215
8.5	Thresholds in terms of $(\text{SNR} \times L)$	217
8.6	Comments	221
8.6.1	Schmidt's Definition of Resolution	221
8.6.2	CRB at the Resolution Threshold	222
8.6.3	Directional Arrays	223
8.7	Array Capabilities based on α and β curves	224
8.8	Summary	226
8.9	Appendices	226
8.9.1	Radius of Circular Approximation	226
8.9.2	'Circular' and 'Y' Arrays - Sensor Locations	229
8.9.3	Cramer Rao Bound in Terms of Principal Curvature . . .	229
	Bibliography	235
	Index	237